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Optical Flow

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This paper deals with computing the optical flow field for a set of image sequences, using differential technique, namely the Horn and Schunck approach ([2]). The images are converted into numbers using (each number in the obtained file represents the intensity of each pixel from the original image) using the software ImageJ, a Java technology. As expected, this model does not give results in real-time, as one can see in the numerical experiments section, but is the cornerstone for the future approaches.

1. Introduction

Optical flow is a motion in two dimensions that needs to be recovered from a sequence of images. This is neccesary to be done in computer vision, medical imaging, video processing, etc. Other points of view consider optical flow being the distribution of apparent velocities of movement of brightness patterns in an image, this arising from relative motion of objects and the viewer (Gibson 1950, 1966), or optical flow is "the apparent motion of brightness patterns in the image " (Robyn Owens 1997), or optical flow is "a concept for considering the motion of objects within a visual representation. Typically the motion is represented as vectors originating or terminating at pixels in a digital image sequence"(Wikipedia-free encyclopedia).

The idea behind the optical flow computation method is the following. Having two subsequent images, we want to compute the vector field which describes the movement of each pixel from the first image to the second one. In order to do this, we transform each image in a matrix of numbers, and then we use the Horn and Schunck method (see [2]). The two considered images must have the same resolution and be part of the same scene from a movie. And it is our attempt to compute the

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optical flow for this two images, and consequently, to obtain an intermediary image, between the two initial ones. If we have a digital image sequence in which objects or even the camera are moving, one can obtain valid informations regarding the scene just by analysing the differences that apear from one image to another, difference caused by this mouvement. If, for example, we have a car moving in a scene, by the analysis of the differences of two consecutive images, one can determine which pixels is form the car, and which are part of the static background. By studying in detail the movement, one can determine: how many objects are in the image, in wich direction are they moving, if the movement is liniar or not, how fast are the objects moving, etc. The most difficult part, though, is the computation of the optical flow field (or image velocity).

2. Model for motion represention

In order to detect the motion (for computing the optical flow field), one can use this classical model. If we denote by $I(x, y, t)$ the measured image intensity at position (x, y, t), considering $I(x, y, t) =$ constant for $t \geq \infty$, we obtain

$$\frac{\mathrm{d}I}{\mathrm{d}t} = 0. \tag{1}$$

Using the chain rule

$$\frac{\mathrm{d}I}{\mathrm{d}t} = \frac{\partial I}{\partial x} \cdot \frac{\mathrm{d}x}{\mathrm{d}t} + \frac{\partial I}{\partial y} \cdot \frac{\mathrm{d}y}{\mathrm{d}t} + \frac{\partial I}{\partial t}$$

we obtain the optical flow equation:

$$\frac{\partial I}{\partial x} \cdot \frac{\mathrm{d}x}{\mathrm{d}t} + \frac{\partial I}{\partial y} \cdot \frac{\mathrm{d}y}{\mathrm{d}t} + \frac{\partial I}{\partial t} = 0. \tag{2}$$

Let us denote by

$$u = \frac{\mathrm{d}x}{\mathrm{d}t}, \quad v = \frac{\mathrm{d}y}{\mathrm{d}t}, I_x = \frac{\partial I}{\partial x}, \quad I_x = \frac{\partial I}{\partial y}.$$

Thus, the equation (2) becomes

$$I_x \cdot u + I_y \cdot v + I_t = 0. \tag{3}$$

Here (u, v) denotes the optical flow field.

Since the motion field is not uniquely determined by (3)(we have two unknowns u and v, and one equation), additional assumptions must be imposed. In this paper we use the variational optical flow computation algorithm proposed in [2]. The authors proposed to minimise the function

$$\Delta^2 u = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}, \; \Delta^2 v = \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}$$

So, we shall solve the problem:

$$\min(\Delta^2 u + \Delta^2 v).$$

2.1. Aproximation of partial derivatives and the Laplacean

For the partial derivatives of the intensity, we use as approximants the following operators:

$$\begin{aligned}
I_x = \; & (I_{i+1,j,k} - I_{i,j,k} + I_{i+1,j+1,k} - I_{i,j+1,k} + I_{i+1,j,k+1} - \\
& - I_{i,j,k+1} + I_{i+1,j+1,k+1} - I_{i,j+1,k+1})/4 \\
I_y = \; & (I_{i,j+1,k} - I_{i,j,k} + I_{i+1,j+1,k} - I_{i+1,j,k} + I_{i,j+1,k+1} - \\
& - I_{i,j,k+1} + I_{i+1,j+1,k+1} - I_{i+1,j,k+1})/4 \\
I_t = \; & (I_{i+1,j,k} - I_{i,j,k} + I_{i+1,j+1,k} - I_{i,j+1,k} + I_{i+1,j,k+1} - \\
& - I_{i,j,k+1} + I_{i+1,j+1,k+1} - I_{i,j+1,k+1})/4.
\end{aligned}$$

For the estimation of the Laplacean, we shall use

$$\Delta u \approx k(\bar{u}_{i,j,k} - u_{i,j,k}), \qquad \Delta v \approx k(\bar{v}_{i,j,k} - v_{i,j,k}).$$

Now, considering $k = 3$, and $\bar{u}$ and $\bar{v}$ will be the local median of u and v, we shall aproximate them by a 9-point stencil scheme, as follows

$$\begin{bmatrix} \frac{1}{6} & \frac{1}{12} & \frac{1}{6} \\ \frac{1}{12} & -1 & \frac{1}{12} \\ \frac{1}{6} & \frac{1}{12} & \frac{1}{6} \end{bmatrix}.$$

This way, we obtain

$$\begin{aligned}
\bar{u}_{i,j,k} = \; & \frac{1}{6}(u_{i-1,j,k} + u_{i,j+1,k} + u_{i+1,j,k} + u_{i,j-1,k}) + \frac{1}{12}(u_{i-1,j-1,k} + u_{i-1,j+1,k} + \\
& + u_{i+1,j+1,k} + u_{i+1,j-1,k}) \\
\bar{v}_{i,j,k} = \; & \frac{1}{6}(v_{i-1,j,k} + v_{i,j+1,k} + v_{i+1,j,k} + v_{i,j-1,k}) + \frac{1}{12}(v_{i-1,j-1,k} + v_{i-1,j+1,k} + \\
& + v_{i+1,j+1,k} + v_{i+1,j-1,k}).
\end{aligned}$$

3. Iterative solution

As iterative solver, we shall use a (coupled) Gauss-Seidel relaxation method. We want to minimise

$$\varepsilon_b = I_x \cdot u + I_y \cdot v + I_t, \;\; \varepsilon_c^2 = (\bar{u} - u)^2 + (\bar{v} - v)^2. \tag{4}$$

By minimising this, we get:

$$\left\{ \begin{array}{llcl} (\alpha^2 + I_x^2 + I_y^2) & (u - \bar{u}) & = & -I_x(I_x\bar{u} + I_y\bar{v} + I_t) \\ (\alpha^2 + I_x^2 + I_y^2) & (v - \bar{v}) & = & -I_y(I_x\bar{u} + I_y\bar{v} + I_t) \end{array} \right. . \tag{5}$$

So, we obtain the iterative scheme

$$\begin{cases} u^{n+1} = \bar{u}^n - \dfrac{I_x(I_x\bar{u}^n + I_y\bar{v}^n + I_t)}{(\alpha^2 + I_x^2 + I_y^2)} \\ v^{n+1} = \bar{v}^n - \dfrac{I_y(I_x\bar{u}^n + I_y\bar{v}^n + I_t)}{(\alpha^2 + I_x^2 + I_y^2)}. \end{cases} \tag{6}$$

In [2], the authors proposed as a stopping criterion the computation of a number of iteration at most equals with the length of the diagonal for the original image.

4. Experiments

The algorithm implemented using the Horn & Shunk approach calculates the optical flow field by computing the partial derivates of the intensity, having the intensity in every pixel for the two images.

Example 1. *We consider as in* [3], *a simple example for the intensity function:* $I(x, y, t) = x + y + t$. *We take* $\alpha = 0.5$ *and the initial approximation* $u_0 = v_0 = 0$. *The resolution of the images is* 65×65. *For this simple example we shall compute only one iteration (we do not need more than one).*

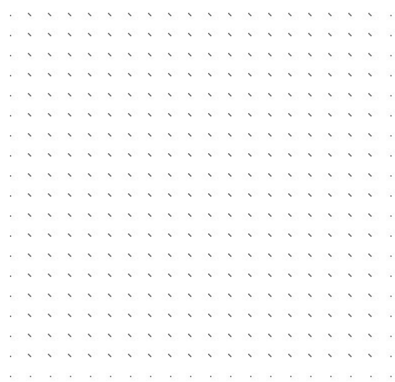

Fig. 1. One of the image sequences (left). Optical flow field after one iteration (right).

Example 2. *We shall consider now the example of a rotating sphere. We considere the set of images presented in* [6] *where we have a rotating sphere around its axe. The resolution of the images is* 200×200*; we took* α *variable for testing what happens with the optical flow field.*

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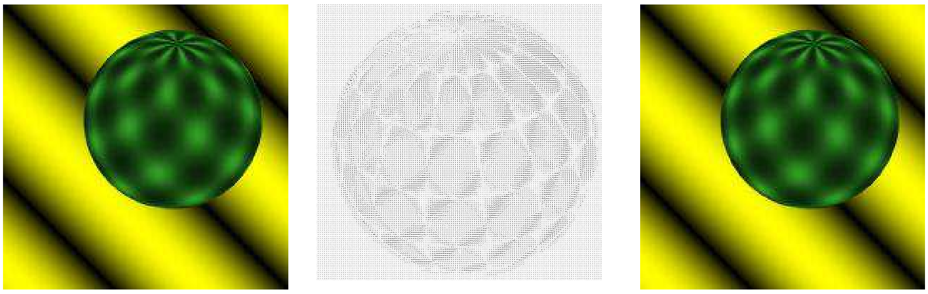

Fig. 2. First image generated after one iteration in the Horn & Schunck algorithm (middle). And the two consecutive images (left and right).

its axe. The resolution of the images is 200×200*; we took* α *variable for testing what happens with the optical flow field.*

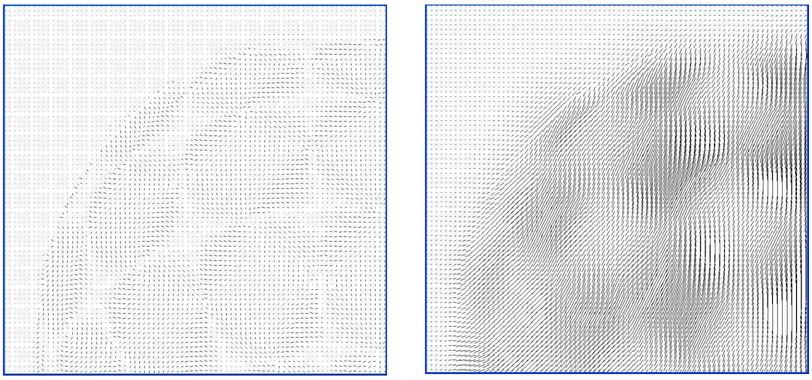

Fig. 3. Optical flow field generated after one iteration with Horn&Schunck algorithm - 750 millisec (left) and after 100 iterations - 53 625 millisec (right).

It can be noticed in Figure 3 the patterns on the sphere (this is not an homogenous vector field) (left image), and an improvement in the optical flow field (right image).

In Figure 2. we can notice, also, an improvement in the optical flow field, but also an influence in the vector field having the gradient (for intensity) equal to 0. This is the moment when the theory tels us to stop.

In this last case, we can notice an improvement in the optical flow field, but also an influence in the vector field having the gradient (for intensity) equal to 0. But there are some problems with the optical flow field from boundary of the sphere, so we should continue no more.

As future work, we want to use different approaches (not only the differential-like techniques) for synthetic and, also, natural images, and to improve the runtime of the implemented algorithms.

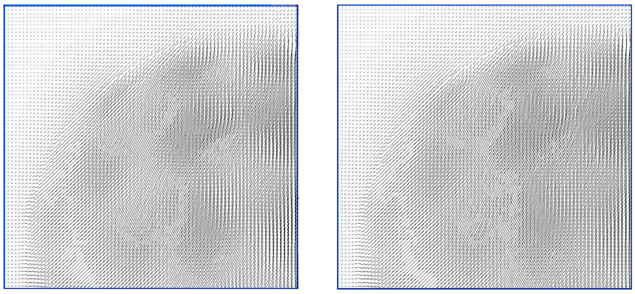

Fig. 4. Optical flow field generated after 200 iterations with Horn&Schunck algorithm – 86 032 millisec (left) and after 300 iterations – 130 219 millisec (right).

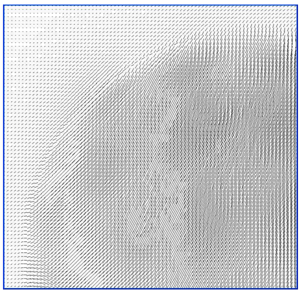

Fig. 5. Optical flow field generated after 400 iterations in the Horn-Schunck algorithm – 182 657 millisec.

Remark 1. *All tests were run on a PC with P4 procesor with* 768 MB *Memory.*

References

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