Constructal Optimization
of Spherical Photovoltaic Cells[♮]

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This paper reports the optimization results on a new generation of photovoltaic cells (PVC), called "spheral", based on constructal theory. In this approach, the optimal shape (geometry) and structure of the photovoltaic ensemble is the outcome of the functionality and constraints that the system (the PVC) has to stand. The cell is assumed to work under steady state conditions, and the electrical optimization is carried out for the maximum power point. The primary unknown in this problem is the electrical potential (voltage), and its distribution is found by numerically integrating the mathematical model of the DC current distribution within the cell, formulated for 2D, in-plane layout. The numerical study is carried out by COMSOL, a multi-physics FEM software package.

1. Introduction

The worldwide photovoltaic (PV) solar energy market has expanded rapidly over the last few years due to the energy crisis that leads to advances in the alternative energy sources. The world solar cell power generation systems installed in 2005 reached 1460 MW, and in the future it is expected to grow [1]. In order to reduce the costs and the consumption of raw materials, innovative manufacturing technologies are developed and introduced to the market, for instance the Spheral Solar Power (SSP) [2] and the spherical photovoltaic cells produced by Kyosemi (Fig. 1) [4].

Spherical Photovoltaic Cells (SPVC) are a novel solar technology that comprises thousands of inexpensive tiny silicon spherical beads arrayed between two aluminum foils. The spheres function like individual solar cells, absorbing sunlight and convert-

ing it into electricity, while the aluminum foils provide the electrical contacts: the front foil serves as the electrical (negative) contact to the n-layer and sets the spacing of the spheres, whereas the back foil functions as the electrical positive contact to the core of the spheres (p-type) (Fig. 1b) [2]. Kyosemi technology relies on wired array of SPV beads, embedded in flexible or rigid glass or silicon tiles [4].
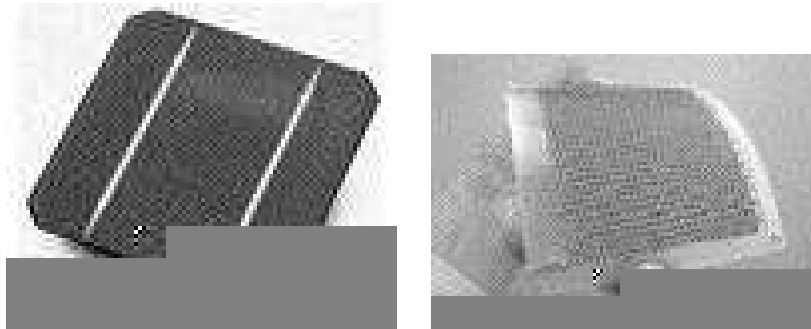
Fig. 1. Polysilicon and spherical solar cells. Conventional, flat polysilicon solar cell [2](left), Spherical PVC [2](right).

The geometrical shape of a solar cell offers a larger sunlight reception surface that can capture light three dimensionally, from virtually all directions, not only as direct sunlight but also as light diffused by clouds and as light reflected from buildings, increasing thus its power generation capacity. SPVCs have great potential because they are inexpensive, simple, and can be ensemble in flexible modules and integrated in new application possibilities.

2. The Constructal Principle

Although the SPV cell is very small (0.2–1 mm), its maximum open voltage is the same as that of a larger flat junction type cell (Fig. 2). Unlike conventional PV cells that are produced by slicing crystalline silicon wafers – a process which generates large losses – the manufacturing process of spherical cells uses recycled silicon from the chip-making industry, fused and shaped into beads. This process involves sphere fabrication, n-p junction formation and finishing, cell fabrication, and assembly of the modules.

When a PVC receives light, electric charges (electron–hole pairs) are generated by PV effect. The majority of the electrical charge is drained across the p-n junction in opposite directions by the action of the electrical field produced at the interface [7]. The accumulations of electrical charges at the external electrodes lead to the buildup of an external voltage between the metal contacts that may then drive the electrical charges through an external electrical load.

The major factors that impair the efficiency of the PVC are: reflection of the
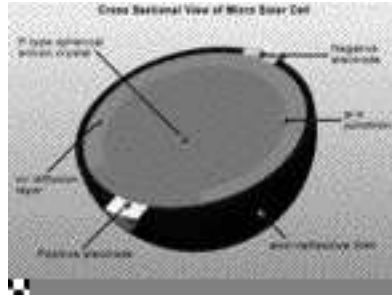
Fig. 2. The structure of a spherical cell.

incident light, its incomplete absorption, the partial utilization of the photons' energy, the partial recombination of the charge carriers and the leakage across the junction [6]–[9]. The PV current is driven, through the n-layer, to a high conductivity grid, up to the electrical terminals: this current path defines the *series resistance* of the PVC, R_S, and it is responsible for the flattening of the current-voltage characteristic and the consequent loss of output power. It can be minimized by using good electrical contacts, surface layers of low resistance, and by optimizing the grid (collector) geometry.

The following basic *volume-to-point access problem* is addressed by the *constructal method* [10]: "given a finite size volume in which heat is generated at every point, which is cooled by a small patch (heat sink) located on its boundary, and a finite amount of high (thermal) conductivity material, determine the optimal distribution of high conductivity material through the given volume such that the highest temperature is minimized."

It was shown [12] that this problem has an equivalent electro-kinetic formulation: *given a finite size volume in which electrical current is generated at every point, which is connected by a small patch (terminal) located on its boundary, and a finite amount of high (electrical) conductivity material, determine the optimal distribution of high conductivity material through the given volume such that the highest voltage drop is minimized.*

In the constructal approach, the basic volume-to-point access problem consists of identifying an optimal PV cell (module) architecture that minimizes the series resistance. Previous work [12] reported the flat-surface PVC constructal optimization, based on the heat transfer – electro-kinetic analogy. The constructal problem presented here is different in that the optimization criterion is the localization of the current source (i.e., the positioning of the SPV beads on the aluminum foil) rather than the distribution of the high conductivity material throughout the current-generating volume. The design objective is then to find an optimal SPV cell-to-cell spacing that minimizes R_S: find the particular pattern of the spherical cell distribution on a highly conductive foil such that the series resistance of the construct is minimized. To identify the optimal layout, several configurations are generated by varying the cell-to-cell spacing between spherical PV beads, which are confined to a finite space. The study

reported next relies on the numerical optimization of the structure and shape of a SPV ensemble.

3.   The Mathematical Model

In what follows, we assume that the SPV system operates under DC conditions and, hence, the electrical field is derived from the electrical potential, V. For a SPV bead with very thin emitter (the n-layer) and collector (Aluminum foil) grid the current flow in the emitter and collector is $2D$. The governing equations that make the physical model are then

Faraday's law

$$\operatorname{curl}\mathbf{E} = 0 \rightarrow \mathbf{E} = -\operatorname{grad}V, \tag{1}$$

Electric charge conservation

$$\operatorname{div}\mathbf{J} = \begin{cases} 0, & (collector) \\ w''', & (emitter) \end{cases} \tag{2}$$

Electrical conduction in the emitter and collector

$$\mathbf{J} = \sigma_{0,P}\mathbf{E}. \tag{3}$$

The quantities in these equations are the electrical field strength, $\mathbf{E}$ (V/m), the current density, $\mathbf{J}$ (A/m^2), and the electrical conductivities of the metallic collector and PV current generator emitter, $\sigma_{0,P}$ (S/m). The mathematical model for the electrical field follows then:

Laplace equation (in the high conductivity collector region)

$$\Delta V = 0, \tag{4}$$

Poisson equation (in the low conductivity, current-generating, emitter, region)

$$\Delta V = \frac{w'''}{\sigma_0}. \tag{5}$$

Equations (4) are non-dimensionalized by dividing the coordinates through a length scale (here, the radius of a SPV bead, r), and the current source through its actual value, $w'''(= \text{const.})$. The voltage scale results then as $V_0 = \frac{LJ_n}{\sigma_0}$. The electrical conductivities, σ_0 and σ_P, are scaled with σ_0.

The following boundary conditions close the mathematical model:

- For the inner surface of the shell (the actual *p-n* interface) a non-homogeneous Neumann condition is used to specify the photovoltaic current source – assumed uniform.

- The normal component of the current density at the external surface of the outer shell (n-layer) of the bead is zero.

- For the contact surface between the bead and the aluminum foil a Dirichlet condition is set, $V = 0$, because the excellent electrical conductivity of the collector suggests an almost equipotential contact.

4.   Simulation Results and Discussion

First, a simplified $2D$ axial-symmetric model is formulated in order to evaluate the current distribution through the outer, n-layer of the SPV bead to the collector – Fig. 3 shows the computational domain for the elemental construct as defined in this work.

The structure of a spherical solar cell device

The $2D$ simplified axial model

$2D$ axial model - the electrical field (dimensional quantities)

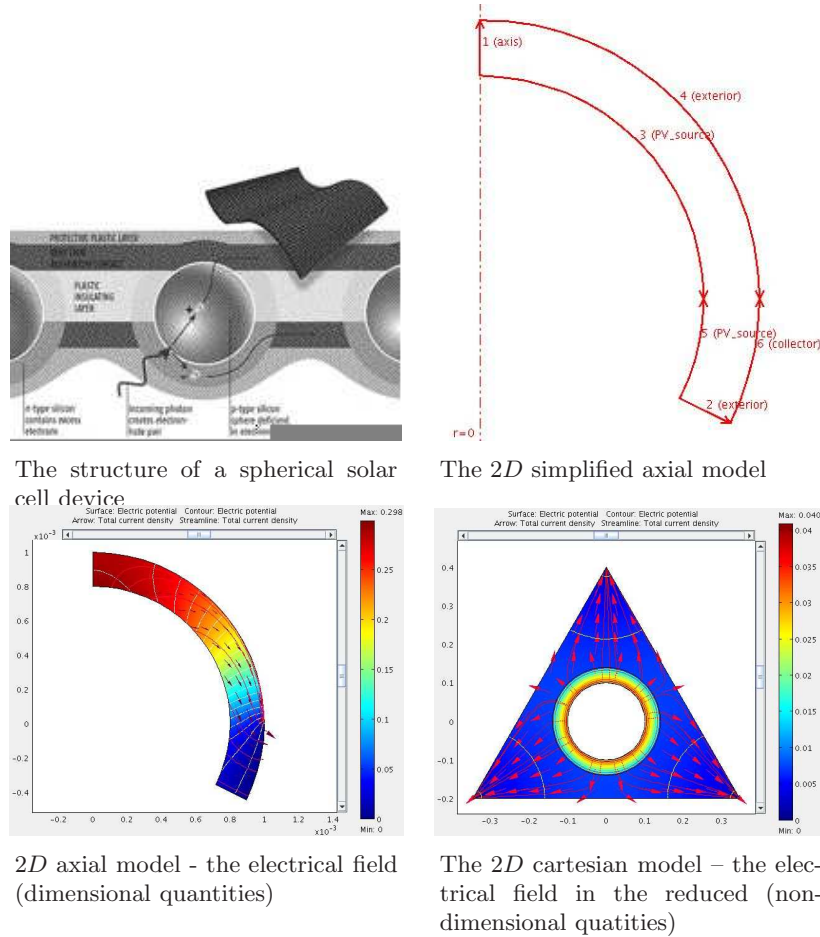The $2D$ cartesian model – the electrical field in the reduced (non-dimensional quatities)

Fig. 3. The 2D simplified models of a spheral solar cell [2] – electrical field spectra.

The numerical accuracy was checked against the current balance: the photovoltaic current is equal to the current delivered at the emitter-collector interface. The $2D$ axial model may be used to evaluate the spherical shell series resistance (n-doped silicon), which makes the (design) series resistance.

The next step is to recast the $2D$ axial model into a $2D$ cartesian model that comprises also the collector (aluminum) "territory" of the bead: the n-shell is pro-

jected onto a circular ring that has the same series resistance as the actual spherical
n-layer; the inner rim of the ring is here the PV current source – it produces the
same amount of current as the inner boundary of the spherical n-shell. As the series
resistance is sensitive to the size of the aluminum patch that embeds the bead, this
quantity may make itself the object of an optimization problem. Figure 3b depicts
the voltage (surface color map and contour lines) and the current flow (arrows and
streamlines) when the external boundary is insulated, except for the vertices that
are set to ground. Of course, symmetry may be used to simplify the solution to the
problem, but the numerical effort to solve this (linear) problem for the entire domain
rather than a subset is not significant.

The $2D$ Cartesian model is further used to define the elemental cell of the
constructal optimization sequence (Fig. 4). The elemental cell may contain a number
of SPV beads, and it is the smallest entity, the "construct" or "brick" that is optimized
for minimum series resistance: its shape and structure are essential to the shape and
structure of higher order constructs in the constructal sequence that follows.

The staggered arrangement of the SPV modules (Fig. 1) suggests the elementary
geometry shown in Fig. 4a, referred to as the *elemental cell* (*EC*) - in constructal
sense. Figure 4b shows meshes produced by the adaptive algorithm used to FEM solve
the conduction problem. Electrical sources are assumed to be uniformly distributed
on the circular interior boundaries and the triangle vertices are patched with tiny
metallized electrodes: they are the current ports to the structure.
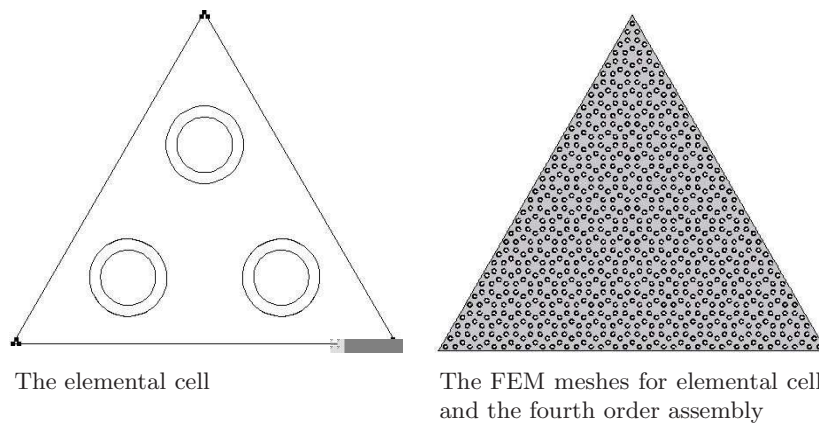

The elemental cell   The FEM meshes for elemental cell
                                      and the fourth order assembly

Fig. 4. Computational domains.

The *design* series resistance of the cell (R_S) – defined as the ratio of the max-
imum potential, $V_{\max}$ through the total current produced by the cell/ensemble, or
"quality factor" (QF), depends on the position of the SPV bead on the triangle axis
(triangle foil). Figure 5 shows the electrical field through the voltage (surface color
map and contour lines) and the current density (arrows).

The optimization of the elemental cell results by evaluating the quality factor
for different structures of *EC*, which differ by the position of the beads centers with

The large spacing limit   The optimal spacing   The small spacing limit
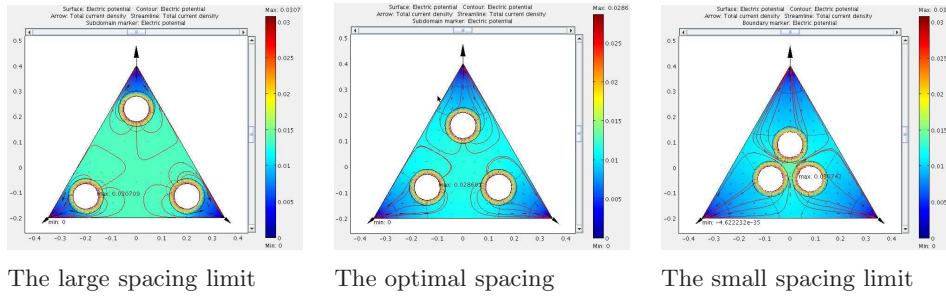
Fig. 5. Constructal optimization sequence for elemental cell.

respect to the mass center of triangle. Figure 5 shows several optimization steps, among which we distinguish the optimal structure - that with the maximum QF. The optimal design, with highest QF (the minimum maximum voltage), is outlined.

In order to pursue with higher order structuring (in the constructal sense) the elemental cell is replicated by mirroring it to its edges, leading thus to the generation of the first, second, third, fourth order constructal ensembles. Figure 6 shows the results obtained by numerical simulation four the first four ensembles generated in this manner.

The ensembles thus generated present almost the same series resistance (the same QF). Figure 6 evidences, in all these instances, a quasi-uniform electrical load – except the vertices regions, which are the electrical terminals.

To certify that the first ensembles thus generated is optimal, an empirical analysis was conducted: we evaluated the series resistance when displacing the beads with respect to their positions produced by reflecting the EC about its edges. We used the symmetry of this design to reduce the computational domain to 1/6 of its actual size, and the beads were displaced such as to preserve symmetry. The analysis confirmed the layout obtained by mirroring the elemental cell as optimal, and the reason is that the aluminum foil has a very good electrical conductivity as compared to that of the beads (n-layer).

The simulation results are presented through the maximum voltage (Fig. 7) and through the overall resistance (Fig. 8), as functions of the conductivities ratio, $\frac{\sigma_P}{\sigma_0}$.

The maximum voltage decreases as the conductivity of the high conductivity material increases, which means that the overall voltage drop due to the series resistance is also a decreasing function of $\frac{\sigma_P}{\sigma_0}$: the higher the conductivity ratio, the lower the voltage-drop on the module, hence the lower the losses by series resistance.

5. Conclusions

The constructal optimization presented in this paper aimed at minimizing the series resistance of the photovoltaic cell/module that ensures the access of the photo-current (internal current) to the electric terminals while meeting a minimal resistance.

First order ensemble.   Second order ensemble

Third order ensemble   Fourth order ensemble
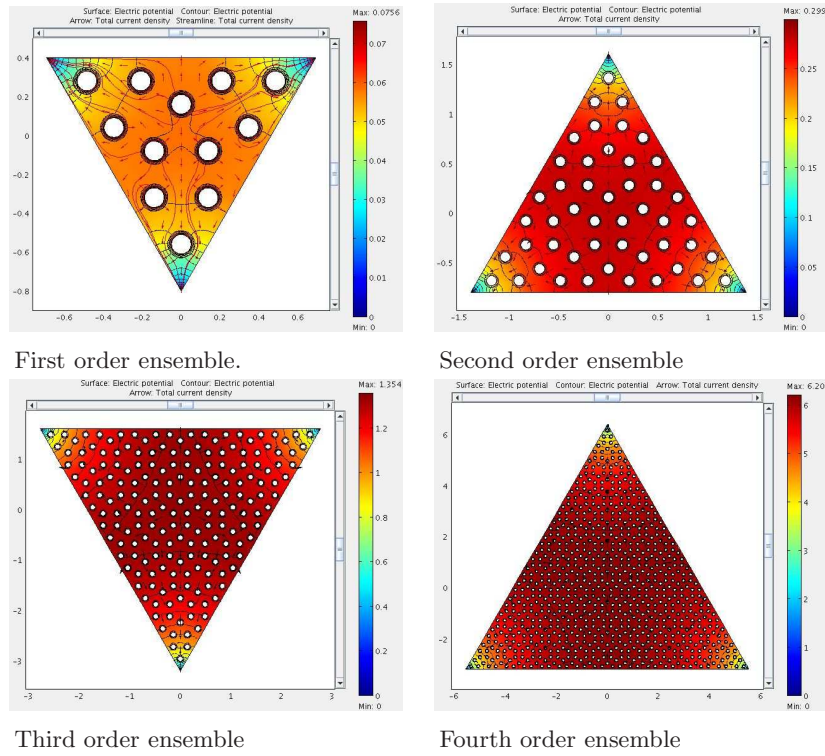
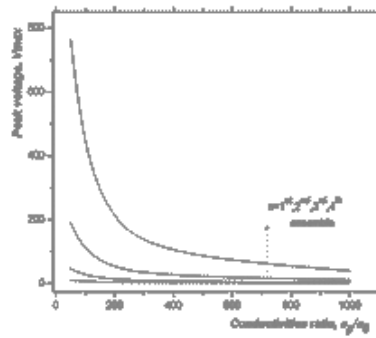Fig. 6. The first four constructal ensembles – the electrical field spectra.

Fig. 7. The peak voltage decreases as the conductivity of the material increases (non-dimensional quantities).

The constructal principle is deterministic, based on physical that describe the phenomena that occur in the investigated systems (here, Maxwell's equations for the electrical conduction).
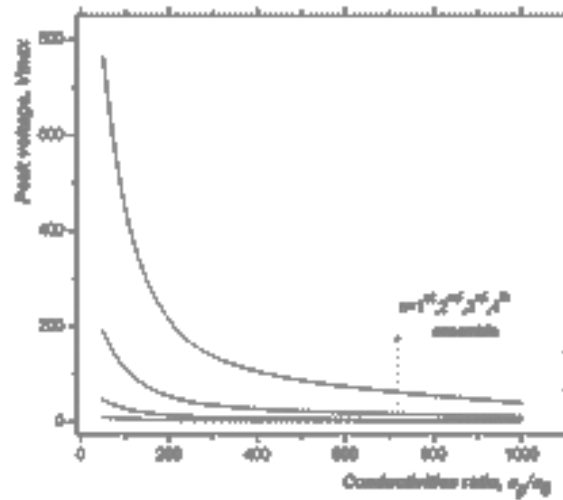
Fig. 8. Series resistance for the first four higher order ensembles
$(n = 1, 2, 3, 4)$ (non-dimensional quantities).

The optimization of the SPVC begins with the elemental cell, and carries on with higher order assembly, from small to large, in a *time-arrow* sequence. This differs significantly from the fractal design, which proceeds from large to small, using consecutive two-way splitting.

As a general rule, the optimized SPV ensembles are built such that their structure provide easier access to the imposed (global) currents that flow through them. The optimized ensembles are constructal, and they possess specific features: the analysis is more and more accurate with increasing conductivities ratio $\frac{\sigma_P}{\sigma_0}$; the higher order ensembles present (almost) same series resistance as the elemental cell.

The constructal principle offers interesting solutions to architectural design goals.

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