Dedicated to Dr. Constantin VĂRŞAN
on the occasion of his 70th birthday

ADAPTIVE CONTROL FOR A CLASS OF STOCHASTIC PASSIVE HOPFIELD NETWORKS

ADRIAN-MIHAIL STOICA and ISAAC YAESH

The paper presents passivity conditions for a class of stochastic Hopfield neural networks with state-dependent noise and with Markovian jumps. Our contributions are mainly based on the stability analysis of the class of stochastic neural networks considered, using infinitesimal generators of appropriate stochastic Lyapunov-type functions. The passivity conditions derived are expressed in terms of the solutions of some specific systems of linear matrix inequalities. The theoretical results are illustrated by a simplified adaptive control problem for a dynamic system with chaotic behaviour.

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1. INTRODUCTION

Hopfield networks are symmetric recurrent neural networks which exhibit motions in the state space which converge to minima of energy.

Symmetric Hopfield networks can be used to solve practical complex problems such as implement associative memory, linear programming solvers and optimal guidance problems. Recurrent networks which are non symmetric versions of Hopfield networks play an important role in understanding human motor tasks involving visual feedback (see [3] and [2] and the references therein). Such networks are subject to effects of state-multiplicative noise, pure time delay (see [7], [10] and [15]) and even multiple attractors which can be caused by Markov jumps. Even without Markov jumps, a non symmetric class of Hopfield networks is able to generate chaos [9]. Therefore, Hopfield networks can be used [14] as identifiers of unknown chaotic dynamic systems. The resulting identifier neural networks have been used in [14] to derive a locally optimal robust controller to remove the chaos in the system.

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In this paper, the robust controller of [14] is replaced by a direct adaptive controller. More specifically, we consider the so called Simplified Adaptive Control (SAC) method [8] which applies a simple proportional controller whose gain is adapted according to the squared tracking error. Since such controllers’ stability proof involves a passivity condition, a passivity result for the Hopfield network is derived. The results presented in this paper are, in fact, developed for a generalized version of non symmetric Hopfield networks including Markov jumps of the parameters and state multiplicative noise, thus allowing a wider stochastic class of chaos generating systems to be considered. They also represent an extension of the problem presented in [17] to the case where the stochastic Hopfield network is passivied via unconstant state-feedback gains.

The paper is organized as follows. In Section 2 the problem is formulated while in Section 3 we derive Linear Matrix Inequalities (LMIs) based conditions for passivity analysis. Section 4 deals with an adaptive control application, Section 5 includes a chaos control example and Section 6 the conclusions.

Throughout the paper, \( \mathcal{R}^{n} \) denotes the \( n \)-dimensional Euclidean space, \( \mathcal{R}^{n \times m} \) is the set of all \( n \times m \) real matrices, and the notation \( P > 0 \) (respectively, \( P \geq 0 \)) for \( P \in \mathcal{R}^{n \times n} \) means that \( P \) is symmetric and positive definite (respectively, semi-definite). Throughout the paper \( (\Omega, \mathcal{F}, P) \) is a given probability space; the argument \( \theta \in \Omega \) will be suppressed. Expectation is denoted by \( E\{\cdot\} \) and conditional expectation of \( x \) on the event \( \theta(t) = i \) is denoted by \( E[x|\theta(t) = i] \).

2. PROBLEM FORMULATION

The neural network proposed by Hopfield can be described by a system of ordinary differential equations of the form

\[
\dot{v}_i(t) = a_i v_i(t) + \sum_{j=1}^{n} b_{ij} g_j(v_j(t)) + \bar{c}_i = \kappa_i(v), \quad 1 \leq i \leq n,
\]

where \( v_i \) is the voltage on the input of the \( i \)th neuron, \( a_i < 0, 1 \leq i \leq n \), \( b_{ij} = b_{ji} \) and the activations \( g_i(\cdot), i = 1, \ldots, n \) are \( C^1 \)-bounded and strictly increasing functions.

This network is usually analyzed by defining the network energy functional

\[
E(v) = -\sum_{i=1}^{n} a_i \int_{0}^{v_i} u \frac{dg_i(u)}{du} du - \frac{1}{2} \sum_{i,j=1}^{n} b_{ij} g_i(v_i) g_j(v_j) - \sum_{i=1}^{n} \bar{c}_i g_i(v_i).
\]
It can be seen that $\frac{dE}{dt} = -\sum \frac{dg(v_i)}{dx_i} \kappa_i(v)^2 \leq 0$, where the zero rate of the energy is only obtained at the equilibrium points, also referred to as attractors, where

$$\kappa_i(v^0) = 0, \quad 1 \leq i \leq n.$$  

The network is then described in matrix form as

$$\dot{v}(t) = Av(t) + Bg(v) + \bar{C}, \quad 1 \leq i \leq n,$$

where

$$A := \text{diag}(a_1, \ldots, a_n), \quad B := [b_{ij}]_{i,j=1,\ldots,n},$$

$$\bar{C} := [\bar{c}_1 \ \bar{c}_2 \ \ldots \ \bar{c}_n]^T, \quad v := [v_1 \ v_2 \ \ldots \ v_n]^T$$

and

$$g(v) := [g_1(v_1) \ g_2(v_2) \ \ldots \ g_n(v_n)]^T.$$  

The stochastic version of this network driven by white noise has been considered in [7] where the stochastic stability of (2.1) has been analyzed and it has been shown that the network is almost surely stable when the condition $\frac{dE}{dt} \leq 0$ is replaced by $L E \leq 0$, where $L$ is the infinitesimal generator associated with the Itô type stochastic differential equation (2.4). This condition has been shown in [7] to be only satisfied in cases where the driving noise in (2.1) is not persistent. This non persistent white noise can be interpreted as a white-noise type uncertainty in $A$ and $B$, namely, state-multiplicative noise. In [16] and [15] Hopfield networks with Markov jump parameters have been considered to represent also nonzero mean uncertainties in these matrices. Encouraged by the insight gained in [3] and [2] regarding the role of state-multiplicative noise and time delay (see also [13]) in visuo-motor control loops, we generalize the results of [16] to include this effect. The Lur’e-Postnikov systems approach ([12], [1]) is invoked to analyze stability and disturbance attenuation (in the $H_\infty$ norm sense), and the results are given in terms of Linear Matrix Inequalities (LMI).

To analyze input output properties we first define the error of the Hopfield network output with respect to its equilibrium points by

$$x(t) = v(t) - v^0.$$  

and assume that the errors vector $x(t)$ satisfy

$$dx(t) = [A_0 (\theta(t)) x(t) + B_0 (\theta(t)) f(y(t)) + D (\theta(t)) u(t)] dt$$

$$+ A_1 (\theta(t)) x(t) d\eta(t) + B_1 (\theta(t)) f(y(t)) d\xi(t),$$

where

$$y(t) = C (\theta(t)) x(t)$$
and the system measured output is
\[(2.8) \quad z(t) = L(\theta(t))x(t) + N(\theta(t))u(t).\]

Note that (2.6) was obtained from (2.4) by replacing \(Adt\) by \(A_0dt + A_1d\eta\), \(Bdt\) by \(B_0dt + B_1gd\xi\) and \(f(x) = g(x + v_0) - g(v_0)\). The control input \(u(t)\) is introduced to provide a stochastic version of [14] allowing the considered Hopfield network to also serve a chaotic system identifier. In [14] the control signal is of the form \(\phi(r)u\) rather than just \(u\), where \(\phi(r)\) is a diagonal matrix having \(f_i(r_i)\) on its diagonal, with \(r = H(\theta(t))x\). For simplicity, here, \(\phi = I\), which also is motivated by the example presented in Section 4.

Note also that the matrices \(A_0(\theta(t)), A_1(\theta(t)), B_0(\theta(t)), B_1(\theta(t)), D(\theta(t)), C(\theta(t))\) and \(L(\theta(t))\) are piecewise constant matrices of appropriate dimensions whose entries are dependent upon the mode \(\theta(t) \in \{1, \ldots, r\}\), where \(r\) is a positive integer denoting the number of possible modes between which the Hopfield network parameters can jump. Namely, \(A_0(\theta(t))\) attains the values of \(A_{01}, A_{02}, \ldots, A_{0r}\), etc. It is assumed that \(\theta(t), t \geq 0\), is a right continuous homogeneous Markov chain on \(D = \{1, \ldots, r\}\) with a probability transition matrix
\[(2.9) \quad P(t) = e^{Qt}, \quad Q = [q_{ij}], \quad q_{ij} \geq 0, \quad i \neq j, \quad \sum_{j=1}^{r} q_{ij} = 0, \quad i = 1, 2, \ldots, r.\]

Given the initial condition \(\theta(0) = i\), at each time instant \(t\), the mode may maintain its current state or jump to another mode \(i \neq j\). The transitions between the \(r\) possible states \(i \in D\) may be the result of random fluctuations of the actual network components (i.e., resistors, capacitors) characteristics or can used to artificially model deliberate jumps which are the result of parameter changes in an optimization problem the network is used to solve. In visuo-motor tasks, one may conjecture that proportional and derivative feedbacks are applied on the basis of time sharing, where transition probabilities define the statistics of switching between the two modes. Although there is no evidence for this conjecture, the model analyzed in the present paper can be used to check its stability and \(L_2\) gain.

In the forthcoming analysis, it is assumed that the components \(f_i, i = 1, \ldots, n\), of \(f(\xi)\) satisfy the sector conditions
\[(2.10) \quad 0 \leq \zeta_i f_i(\zeta_i) \leq \zeta_i^2 \sigma_i,\]
which are equivalent to
\[(2.11) \quad -F_i(\zeta_i, f_i) := f_i(\zeta_i)(f_i(\zeta_i) - \sigma_i \zeta_i) \leq 0.\]

It is further assumed that
\[(2.12) \quad \frac{df_i}{d\zeta_i} \leq \sigma_i, \quad i = 1, \ldots, n,\]
which is not restrictive since it is fulfilled by the usual nonlinearities as saturation, sigmoid, etc., used in the neural networks.

As mentioned above, the passivity (in stochastic sense) condition for systems (2.6)–(2.8) which is expressed as

\[ J = E \left\{ \int_{0}^{\infty} (z^T(t)u(t)) \, dt \right\} > 0, \quad x(0) = 0, \]

will be further analyzed.

3. PASSIVITY ANALYSIS

Introduce the Lyapunov-type function

\[ V(x(t), \theta(t)) = x^T(t) P(\theta(t)) x(t) + 2 \sum_{k=1}^{n} \lambda_k \int_{0}^{C^{(k)}(\theta(t))x} f_k(s) \, ds \]

depending on the nonlinearities \( f_k(y_i) = f_k(C^{(k)}_i x) \) via the constants \( \lambda_k, k = 1, \ldots, n \), where \( C^{(k)}_i \) is the kth row in \( C_i \), \( i = 1, \ldots, r \). As it was mentioned in [1], \( V \) of (3.14) defines a parameter-dependent Lyapunov function. Indeed, in the special case of \( f_i(x_i) = \sigma_i x_i \) one gets \( V(x, \sigma_1, \sigma_2, \ldots, \sigma_n) = x^T(P + S^\frac{1}{2} \Lambda S^\frac{1}{2})x \), where the notation

\[ S := \text{diag} (\sigma_1, \sigma_2, \ldots, \sigma_n) \quad \text{and} \quad \Lambda := \text{diag} (\lambda_1, \lambda_2, \ldots, \lambda_n) \]

has been introduced.

Using the Itô-type formula (see [4], [5] and [6]) for \( V(x, \theta) \) we get

\[ E \{ V(x, \theta(t))|\theta(0) \} - E \{ V(0, \theta(0)|\theta(0) \} = E \left\{ \int_{0}^{t} \mathcal{L} V(x, \theta(s)) \, ds \right\} \]

with the infinitesimal operator \( \mathcal{L} \) given by

\[ \mathcal{L} V(x, \theta) := (A_0(\theta) x + B_0(\theta) f(y) + D(\theta) u)^T \frac{\partial V}{\partial x} + x^T A^T_1(\theta) P A_1(\theta) x + f^T B^T_1(\theta) P B_1(\theta) f + \sum_{j=1}^{r} q_{ij} x^T P_j x, \]

where

\[ \bar{P}(\theta, \lambda_1, \lambda_2, \ldots, \lambda_n) : = P(\theta) + \text{diag} \left( \lambda_1 \frac{\partial f_1}{\partial x_1}, \ldots, \lambda_n \frac{\partial f_n}{\partial x_n} \right) \quad \text{and} \quad f := f(y(t)). \]

Here, the dependence on the argument \( t \) was omitted for simplicity. Then condition (2.13) is fulfilled if

\[ \mathcal{L} V < 2z^T u, \]
which becomes

\[
\begin{align*}
(x^T A_{0i}^T + f^T B_{0i}^T + u^T D_i^T) (P_i x + C_i^T \Lambda f) + \\
+ (x^T P_i + f^T \Lambda C_i) (A_{0i} x + B_{0i} f + D_i u) + \\
x^T A_{1i}^T P_i A_{1i} x + f^T B_{1i}^T P_i B_{1i} f + \sum_{j=1}^r q_j x^T P_j x - \\
u^T L_i x - x^T L_i^T u - u^T (N_i + N_i^T) u < 0, \quad i = 1, \ldots, r,
\end{align*}
\]

where \( \bar{P}_i \) denotes \( \bar{P}(\theta = i, \lambda_1, \lambda_2, \ldots, \lambda_n) \).

In order to explicitly express (3.18), assumption (2.12) will be used. Indeed, using inequalities (2.12) we see that conditions (3.18) are fulfilled if (3.19)

\[
F_{i0}(x, f) := x^T \left[ A_{0i}^T P_i + P_i A_{0i} + A_{1i}^T \left( P_i + C_i^T S^2 \Lambda S^2 C_i \right) A_{1i} + \sum_{j=1}^r q_j P_j \right] x + \\
+ u^T D_i^T C_i^T \Lambda f + f^T \Lambda C_i D_i u + f^T \left[ B_{0i}^T C_i^T \Lambda + \Lambda C_i B_{0i} \right] + \\
+ B_{1i}^T \left( P_i + C_i^T S^2 \Lambda S^2 C_i \right) B_{1i} \right] f + f^T \left( B_{0i}^T P_i + \Lambda C_i A_{0i} \right) x + \\
x^T (P_i B_{0i} + A_{0i}^T C_i^T \Lambda) f - u^T (L_i - D_i^T P_i) x - \\
x^T (L_i^T - P_i D_i) u - u^T (N_i + N_i^T) u < 0.
\]

Using the \( S \)-procedure (e.g., [1]), we deduce that (3.17) subject to (2.11) is satisfied if there exist \( \tau_i \geq 0, \quad i = 1, 2, \ldots, n \), such that

\[
F_{i0}(x, f) - \sum_{k=1}^n \tau_k F_k(x, f) \geq 0.
\]

Denoting

\[
T := \text{diag}(\tau_1, \tau_2, \ldots, \tau_n)
\]

and noticing that

\[
- \sum_{k=1}^n \tau_k F_k(x, f) = \sum_{k=1}^n \tau_k f_k^2 - \tau_k \sigma_k f_k y_k = f^T T f - \frac{1}{2} f^T T C_i S x - \frac{1}{2} x^T S C_i^T T f,
\]

one gets from (3.20) that

\[
x^T Z_{i11} x + f^T Z_{i12} x + x^T Z_{i12} f + f^T Z_{i22} f - u^T (L_i - D_i^T P_i) x + u^T D_i^T C_i^T \Lambda f \\
+ f^T \Lambda C_i D_i u - x^T (L_i^T - P_i D_i) u - u^T (N_i + N_i^T) u < 0, \quad i = 1, \ldots, r.
\]
where

\[
Z_{i11} := A_0^T P_i + P_i A_{0i} + \sum_{j=1}^{r} q_{ij} P_j,
\]

\[
Z_{i12} := P_i B_{0i} + A_0^T C_i^T \Lambda + \frac{1}{2} S C_i^T T,
\]

\[
Z_{i22} := B_{0i}^T C_i^T \Lambda + A C_i B_{0i} + B_{1i}^T \hat{P}_i B_{1i} - T,
\]

with

\[
\hat{P}_i := P_i + C_i^T S_i^2 \Lambda S_i^2 C_i.
\]

These conditions are fulfilled if

\[
\begin{bmatrix}
Z_{i11} & Z_{i12} & P_i D_i - L_i^T \\
Z_{i12}^T & Z_{i22} & A C_i D_i \\
D_i^T P_i - L_i & D_i^T C_i^T \Lambda & -(N_i^T + N_i)
\end{bmatrix} < 0, \quad i = 1, \ldots, r,
\]

with the unknown variables \(P_i, \Lambda\) and \(T\).

The above developments can be stated as

**Theorem 1.** System (2.6)–(2.8) is stochastically stable and strictly passive if there exist symmetric matrices \(P_i > 0, i = 1, \ldots, r\), and diagonal matrices \(\Lambda > 0\) and \(T > 0\) satisfying system (3.24) of LMIs.

4. SIMPLIFIED ADAPTIVE CONTROL

In this section it is shown how a stochastic system of the form (2.6)–(2.7) can be regulated using a direct adaptive controller of the type

\[
u = -K z,
\]

where

\[
\hat{K} = z z^T.
\]

This type of adaptive control is well-known in the deterministic case (see, e.g., [8]) and in the present section we analyze the particularities arising in the stochastic framework (see also [18]). Throughout this section it is assumed that \(C_i = 0\), which corresponds to the situation when the nonlinearity \(f\) is missing, or \(C_i D_i = 0, i = 1, \ldots, r\). Under this assumption, system (2.6)–(2.7) with \(N_i = \epsilon I\) for \(\epsilon\) tending to zero satisfies the strictly passivity condition (3.24) if there exist symmetric matrices \(P_i > 0, i = 1, 2, \ldots, r\), satisfying the conditions

\[
\begin{bmatrix}
Z_{i11} & Z_{i12} \\
Z_{i12} & Z_{i22}
\end{bmatrix} < 0
\]
and
\[(4.28)\quad P_iD_i = L_i^T, \quad i = 1, 2, \ldots, r,\]
(see e.g. [18] and [8]). The stochastic closed-loop system obtained from (2.6), and (2.7) with \(u = Kz\) can be written as:
\[(4.29)\quad dx = [(A_0(\theta) - D(\theta)K_rL(\theta))x + B_0(\theta)f(y) + D(\theta)\bar{u}]\,dt
+ A_1(\theta)x\,d\eta + B_1(\theta)f(y)d\xi, \quad z = L(\theta)x,\]
where \(\bar{u} := - (K - K_\varepsilon)z\). The above equations hold for any \(K_\varepsilon\) of appropriate dimensions but it is assumed that \(K_\varepsilon\) is such that system (4.29) is stochastically passive (in this case some authors call the open-loop system almost passive (AP)). Note that the existence of \(K_\varepsilon\) is needed just for stability analysis, but its value is not used in the implementation. In the present context, the stochastic stability of this direct adaptive controller (4.25), (4.26) (which is usually referred to as simplified adaptive control-SAC) will be guaranteed by the stochastic version of the AP property. Two cases will be emphasized in what follows.

Case a): \(K_\varepsilon\) is constant. As in [8], to prove the closed-loop stability, we will choose a generalization of the Lyapunov function of (3.14), namely
\[(4.30)\quad \mathcal{V}(x(t), K(t), \theta(t)) = \mathcal{V}(x(t), \theta(t)) + \text{tr}\,(K(t) - K_\varepsilon)^T(K(t) - K_\varepsilon),\]
where \(\text{tr}\) denotes the trace and \(\mathcal{V}\) is given by (3.14) with \(P_i, i = 1, \ldots, r,\) satisfying conditions of the form (4.27) and (4.28) written for the passive system (4.29) relating \(\bar{u}\) and \(z\). Then, denoting \(\bar{A}_0i := A_0i - D_iK_rL_i,\) the infinitesimal generator of \(\mathcal{V}\) of the form (4.30) along the trajectory (4.29) has the expression
\[(4.31)\quad \mathcal{L}\mathcal{V}(x(t), K(t), \theta(t) = i) = 2 \left[ \bar{A}_0i x + B_0if + D_i\bar{u} \right]^T \left( P_i x + C_i^T\tilde{A}_f \right)
+ x^T A_i^TP_iA_i x + \sum_{j=1}^r q_{ij} x^TP_j x + f^TB_i^TP_i f + 2 \text{tr}\,(\bar{K}(K - K_\varepsilon)^T)
= x^T \left( \bar{A}_0i P_i + \bar{P}_i\bar{A}_0i + \bar{A}_1^TP_i A_1i + \sum_{j=1}^r q_{ij} P_j \right) x + f^T \left( B_0i^TP_i + \Lambda C_i\bar{A}_0i \right) x
+ x^T \left( P_iB_0i + \bar{A}_0i^TC_i^T\Lambda \right) f + \bar{u}^TD_i^TP_i x + x^TP_iD_i\bar{u} + \bar{u}^TD_i^TC_i^T\tilde{A}_f f + f^T\Lambda C_iD_i\bar{u} + f^T \left( B_0i^TC_i^T\Lambda + \Lambda C_iB_0i + B_1i^TP_iB_1i \right) f + 2 \text{tr}\,(\bar{K}(K - K_\varepsilon)^T)
= \begin{bmatrix} x^T & f^T \end{bmatrix} \begin{bmatrix} \bar{Z}_{i11} & \bar{Z}_{i12} & x \\ \bar{Z}_{i12} & \bar{Z}_{i22} & f \end{bmatrix} + \bar{u}^TD_i^TP_i x + x^TP_iD_i\bar{u}
+ \bar{u}^TD_i^TC_i^T\tilde{A}_f f + f^T\Lambda C_iD_i\bar{u} + 2 \text{tr}\,(\bar{K}(K - K_\varepsilon)^T),\]
where

\[
\begin{align*}
\tilde{Z}_{i11} &:= \tilde{A}_0^T P_i + P_i \tilde{A}_0 + A_{ii}^T \tilde{P}_i A_{ii} + \sum_{j=1}^{r} q_{ij} P_j, \\
\tilde{Z}_{i12} &:= P_i B_0 + \tilde{A}_0^T C_i^T \Lambda, \\
\tilde{Z}_{i22} &:= B_0^T C_i^T \Lambda + \Lambda C_i B_0 + B_{ii}^T \tilde{P}_i B_{ii},
\end{align*}
\]

(4.32)

with \( \tilde{P}_i \) defined by (3.23). Taking into account that \( P_i D_i = L_i^T, \bar{u} = -(K - K_e)z \) and the assumption \( C_i D_i = 0, \) it follows from (4.31) that

\[
\mathcal{L} V(x(t), K(t), \theta(t) = i) = \begin{bmatrix} x^T & f^T \end{bmatrix} \begin{bmatrix} \tilde{Z}_{i11} & \tilde{Z}_{i12} \\ \tilde{Z}_{i12}^T & \tilde{Z}_{i22} \end{bmatrix} \begin{bmatrix} x \\ f \end{bmatrix} + 2 \text{tr} \left( \dot{K} - zz^T \right) (K - K_e)^T.
\]

(4.33)

For \( \dot{K} = zz^T \) we have

\[
\mathcal{L} V(x(t), K(t), \theta(t) = i) = \begin{bmatrix} x^T & f^T \end{bmatrix} \begin{bmatrix} \tilde{Z}_{i11} & \tilde{Z}_{i12} \\ \tilde{Z}_{i12}^T & \tilde{Z}_{i22} \end{bmatrix} \begin{bmatrix} x \\ f \end{bmatrix}.
\]

(4.34)

Since system (4.29) satisfies the passivity condition (3.24), we get

\[
\begin{bmatrix} \tilde{Z}_{i11} & \tilde{Z}_{i12} \\ \tilde{Z}_{i12}^T & \tilde{Z}_{i22} \end{bmatrix} < 0,
\]

(4.35)

By repeating the \( \mathcal{S} \)-procedure arguments that follow (3.20), we deduce that if (4.35) is fulfilled then

\[
\begin{bmatrix} x^T & f^T \end{bmatrix} \begin{bmatrix} \tilde{Z}_{i11} & \tilde{Z}_{i12} \\ \tilde{Z}_{i12}^T & \tilde{Z}_{i22} \end{bmatrix} \begin{bmatrix} x \\ f \end{bmatrix} < 0
\]

for \( x \) and \( f \) satisfying (2.11). Thus, \( \mathcal{L} V(x(t), K(t), \theta(t) = i) < 0 \), which proves the stochastic stability of the resulting closed-loop system.

**Case b)**: \( K_e \) is time-varying. In this case one can use a control law of the form (4.25) with the modified adaptive gain

\[
\dot{K} = zz^T + \dot{K}_e
\]

(4.36)

but in this case, instead of the constant positive definite matrices in the expression (3.14) of the Lyapunov function, one must use the uniformly positive definite matrices \( P_i(t), t \geq 0, i = 1, \ldots, r, \) satisfying the system

\[
\begin{bmatrix} \dot{P}_i + \tilde{Z}_{i11} & \tilde{Z}_{i12} \\ \tilde{Z}_{i12}^T & \tilde{Z}_{i22} \end{bmatrix} < 0, \quad i = 1, \ldots, r,
\]

(4.37)
where $\tilde{Z}_{11i}$, $\tilde{Z}_{12i}$, and $\tilde{Z}_{22i}$ are given by (4.32). The existence of such positive definite matrices $P_i(t)$, $i = 1, \ldots, r$, together with the diagonal matrix $\Lambda > 0$, is guaranteed by the fact that $K_e(t)$ was assumed to be such that the closed loop system (4.29) is passive. The implementation of the adaptive control law with $K$ given by (4.36) requires in this case the knowledge of the function $K_e(t)$, $t \geq 0$.

We next apply this result to a chaos control problem.

5. EXAMPLE – CHAOS CONTROL

Consider a slightly modified version of the third order chaos generator model of [9] described by (2.6)–(2.7), where

$$
\begin{align*}
A_0 &= \begin{bmatrix} -\epsilon & 1 & 0 \\ 0 & -\epsilon & 1 \\ a_1 & a_2 & a_3 \end{bmatrix}, & B_0 = D = L^T &= \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \\
A_1 &= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & \sigma \\ 0 & 0 & 0 \end{bmatrix}, & B_1 &= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, & C^T &= \begin{bmatrix} \beta \\ 0 \\ 0 \end{bmatrix},
\end{align*}
$$

(5.1)

with $a_1 = -2$, $a_2 = -1.48$, $a_3 = -1$, $\sigma = 0.1$, $\epsilon = 0.01$ and $\beta = 10$. The nonlinearity is $f(y) = \alpha \tanh(y)$, where $\alpha = 1$.

To establish stability, we verify (4.27)–(4.28) with $K_e = 10^5$ using YALMIP [11] where $A_0$ is replaced by $A_0 - DK_e L$. Therefore, by the results of Section 4, the closed-loop system with controller (4.25), (4.26) is expected to be stochastically stable.

Next, we simulate the above system for 500 sec with an integration step of 0.001 s with $u = 0$ for $t \leq 250$ s and with the SAC controller $u = -Kz$, where $\dot{K} = z^2$ in the rest of the time. The results are given in Figures 1–3: the phase-plane (i.e., $x_1$ versus $x_2$) trajectories are depicted in Figure 1, the components $x_i$, $i = 1, 2, 3$, of the state-vector and the control input are depicted in Figure 2 while the adaptive gain $K$ is depicted in Figure 3. It is seen from these figures that the chaotic behavior characterizing the system in open-loop, is replaced by a stable trajectory at $t \geq 250$ s, where the SAC is applied.

6. CONCLUSIONS

A class of stochastic Hopfield networks subject to state-multiplicative noise where the network weights jump according a Markov chain process has been considered. Stochastic passivity conditions for such systems have been derived in terms of Linear Matrix Inequalities. The results have been illustrated via simplified adaptive control of a dynamic system which exhibits a
chaotic behavior when it is not controlled. The control efficiency in stabilizing the chaotic process has been demonstrated by simulations. The results of this paper should encourage further study of attempts to control chaotic systems with simplified adaptive controllers.
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“Politehnica” University of Bucharest
Faculty of Aerospace Engineering
Str. Polizu, No. 1, 011061, Bucharest, Romania
amstoica@rdslink.ro

and

Control Department, IMI Advanced Systems Div.
P.O.B. 1044/77
Ramat-Hasharon, 47100, Israel
iyaesh@imi-israel.com