M. Falcitelli, S. Ianuş and A.M. Pastore, *Riemannian Submersions and Related Topics*, World Scientific, Singapore, 2004, XIII+277 pp., ISBN 981-238-896-6

Riemannian submersions are a constant object of study in differential geometry since the seminal papers of O'Neill (1966) and Gray (1967), where their fundamental equations where established, in an attempt to dualize the theory of Riemannian immersions. The examples are abundant and come from a variety of fields. Maybe the most basic example is the Hopf fibration, a model for the Boothby-Wang fibration of a compact Sasakian manifold over a Hodge manifold. Indeed, an important class of examples comes from principal bundles endowed with compatible metrics. Other examples appear in the context of quaternionic manifolds (the twistor fibration), of homogeneous manifolds, of harmonic maps etc. Important cases appear from physical theories, for example in Kaluza-Klein theories.

There are several criteria according to which one may attempt to classify Riemannian submersions. Among them: the geometric properties of the fibres (totally geodesic, totally umbilical), the structures on the total space and on the base and their relations through the submersion etc. The tools for classifications combine differential geometric tensor calculus and topological methods.

These and many others are the problems raised by the book under review. The contents is devoted to the following topics: Riemannian submersions (introductory chapter, containing many classical examples and the fundamental equations), submersions with totally geodesic fibres (containing mainly Escobales' classification when the total space is a sphere), almost Hermitian submersions (dealing mainly with Watson's results and a glimpse to Marrero and collaborator's result for the locally conformal Kaehler case), Riemannian submersions and contact metric manifolds (presenting mainly submersions from Sasakian or 3-Sasakian manifolds, but also generalizations to f-structures, a subject the second and third named authors wrote a lot lately), Einstein spaces and Riemannian submersions (presenting the canonical variation of Brard-Bergery, Einstein metrics on principal bundles and some material about Einstein-Weyl structures), Riemannian submersions and submanifolds (presenting the notion of CR-submersion as introduced by Kobayashi, and giving the relations between submanifolds of Sasakian and Kaehler manifolds through the Boothby-Wang fibration), semi-Riemannian submersions (presenting results by Magid and classification theorems for the case of totally umbilic fibres by Baditoiu and Ianus), applications of Riemannian submersions in physics (the focus is on the Kaluza-Klein Ansatz and applications).

This is the first monograph entirely devoted to the subject (which appeared till now only as a chapter in Besse's book). Well written, gathering information spread in a lot of papers, unifying the style of many authors, with most of the proofs carried in all details, with a wealth of examples, it certainly feels a gap in the literature and will be a prior reference for both researchers and students.

Liviu Ornea

REV. ROUMAINE MATH. PURES APPL., **52** (2007), *5*, 591–609

M.M. Rao, Editor, *Real and Stochastic Analysis – New Perspectives*, Trends in Mathematics, Birkhäuser, Boston-Basel-Berlin, 2004, VIII+405 pp., ISBN 0-8176-4332-x

This book contains six research-expository articles in stochastic analysis and functional analytic generalizations of probabiliy theory: *Stochastic differential equations and hypoelliptic operators* by Denis R. Bell, *Curved Wiener space analysis* by Bruce K. Deiver, *Noncommutative probabiliy and applications* by Stanley Gudder, *Advances and applications of the Feynman integral* by Brian Jefferies, *Stochastic differential equations based on Lévy processes and stochastic flow of diffeomorphisms* by Hiroshi Kunita, *Convolutions of vector fields*-III: *Amenability and spectral properties* by M.M. Rao.

The first article is devoted to Malliavin's stochastic calculus and its applications to SDEs and hypoelliptic operators. After an introduction to the general theory, the author presents some of his joint results with S. Mohammed concerning degenerate operators.

The second article develops the stochastic calculus on Riemannian manifolds. It starts with a detailed introduction to the differential geometric notions having in mind the stochastic calculus ideas which are introduced later on. One should say that the stochastic calculus on Riemannian manifolds is based on the reinterpretation of some fundamental geometric notions from a new perspective, and that the intuition plays a major role in this construction. Most books on differential geometry strive to explain the formal definitions, letting the reader to understand the intuitive background only with time, after extended readings of different sources. Here is the valuable contribution of the present article, which offers details on the intuition behind the theory, helping the reader which is less familiar with Riemannian geometry to enter the subject of stochastic calculus on manifolds. The article illustrates the scope of stochastic differential geometry with the applications to heat kernels, the study of path spaces and Malliavin's calculus on manifolds.

The third article is a survey of various noncommutative probabiliy theories and their significance to quantum mechanics.

The fourth article is also devoted to operator theoretic generalizations of probabilistic notions with applications to quantum mechanics.

The fifth article contains an exposition of the basic facts about SDEs with jumps. The main topic discussed is the flow of homeomorphisms or diffeomorphisms generated by a SDE. The results presented in the article complements the author's older results for continuous SDEs.

The sixth article concerns random walks on locally compact groups and the spectral theory of the associated convolution operators. The article contains the complete solution of an old problem of M.M. Day.

All the articles provide detailed proofs and discussions of related works with many references. So, this new kind of publication should undoubtedly become of great value to the mathematical community.

L. Stoica

P. Cannarsa and C. Sinestrari, *Semiconcave Functions, Hamilton-Jacobi equations and Optimal Control*, Progress in Nonlinear Differential Equations and Their Applications, Vol. 58, Birkhäuser, Basel-Boston-Berlin, 2004, XIV+304 pp., ISBN 0-8176-4336-2

Initially semiconcave functions were used to obtain global existence and uniqueness results for nonlinear partial differential equations. The progress made in the study of optimization problems, in the last two decades especially, emphasized the importance of semiconcavity for the optimal control theory. The aim of this book is to provide a comprehensive and self-contained presentation of the notion of semiconcavity and of its applications to optimal control problems.

We can divide the book into three parts. In Chapters 2, 3 and 4 the theory of semiconcave functions is developed. Chapter 5 is devoted to the study of Hamilton-Jacobi equations. One of the motivations for introducing this chapter is the close relation between Hamilton-Jacobi equations and the calculus of variation and optimal control problems. The last three chapters treat applications of semiconcave functions to calculus of variations and optimal control theory.

In each part, the authors present classical as well as recent results. Examples and counterexamples are carefully selected. Every chapter ends with bibliographical notes which may guide further reading.

The first chapter has an introductory character and its scope is to emphasize the relationship between semiconcave functions with linear modulus, the calculus of variations, dynamic programming, and Hamilton-Jacobi equations. This is done by studying a simple variational problem.

The notion of semiconcave function with general modulus is introduced in the second chapter. General properties of semiconcave functions, such as their locally Lipschitzianity, are proved. One also studies properties which are specific to semiconcave functions with linear modulus. A few examples are given and one shows how the semiconcavity of the solutions of the parabolic Hamilton-Jacobi equation $\partial_t u(t,x) + |\nabla u(t,x)|^2 = \Delta u(t,x)$ can be exploited to obtain differential Harnack estimates for the solutions of the heat equation. Finally, the almost equivalence between a general semiconcavity estimate and semiconcavity is established. Basic results on convex sets and convex functions needed for a better understanding of the definitions and results of this chapter are presented in Appendix A.1.

Generalizations of the differential and derivatives of a function such as the semi-differentials (superdifferential and subdifferential), the upper and lower Dini derivatives, the reachable gradients, the generalized lower derivatives and Clarkes gradient are defined and studied in Chapter 3. These notions have a crucial importance in obtaining existence and uniqueness results for Hamilton-Jacobi equations, variational calculus and optimal control problems. A special attention is paid to the semi-differentials of a semiconcave function and to those of marginal functions. Inf-convolutions are also defined as a particular case of marginal functions.

Chapter 4 deals with the singularities of semiconcave functions. One proves first, using general results on Hausdorf measures, that the singular set of a semiconcave function defined on an open set $\Omega \subset \mathbb{R}^n$ is $n-1$ rectifiable (the notion of rectifiable set is provided in Appendix A.3). Then a more detailed description of the singular set of a semiconcave function is obtained. Sufficient criteria for the propagation of singularities of semiconcave functions along Lipshitz arcs or, more generally, along k dimensional sets are given.

Chapter 5 is dedicated to Hamilton-Jacobi equations. Since the subject is huge, the authors contented themselves to include here only the results relevant for the other topics discussed in the book. Also, they only provide proofs when they consider them relevant for their scope. The method of characteristics for Hamilton-Jacobi equations and general first order partial differential equations is described in the first section of the chapter. Next, some theorems on viscosity solutions of Hamilton-Jacobi equations are collected and the relation between viscosity solutions and semiconcavity is analyzed. The chapter ends with the study of the propagation of singularities of the solutions of Hamilton-Jacobi equations. Some results from the previous chapter are improved and the propagation of singularities along generalized characteristics is proved.

The notions presented in Chapters 2-5 are applied in the last three chapters of the book to problems arising in calculus of variations and in optimal control. In Chapter 6 the following problem from the calculus of variations is considered. Let Ω be an open set in the n dimensional Euclidean space, $T > 0$, $S_0, S_T \subset \overline{\Omega}$ two closed sets, $\mathcal{A}$ - the set of absolutely continuous arcs $\xi : [0,T] \to \overline{\Omega}$ having ending points in S_0 and S_T. One requires to minimize the functional $\Lambda(\xi) + u_0(\xi(0)) + u_T(\xi(T))$, where $\Lambda(\xi) = \int_0^T L(s, \xi(s), \dot{\xi}(s))\, \mathrm{d}s$ and $L : [0,T] \times \overline{\Omega} \times \mathbb{R}^n \to \mathbb{R}$, $u_0, u_T : \overline{\Omega} \to \mathbb{R}$ are continuous functions. Sufficient conditions for the existence of the minimizers are given. Lipschitzianity of the minimizers (if some additional conditions are satisfied) is obtained and Euler-Lagrange equations are derived. Starting with Section 3, the problem with initial endpoint free is studied. The Hamilton system and value function associated to the problem are introduced. Among the main results proved in this part we mention those regarding the regularity of the value function and the rectifiability of the closure of the set of irregular poins of the problem.

Two optimal control problems with finite time horizon and unrestricted state space, namely the Mayer and Bolza problems, are the topic of the seventh chapter. More attention is payed to Mayer problem, due to the similarities existing between these two problems. Again, after obtaining sufficient conditions for the existence of an optimal control for the problem, the emphasis is on the study of the value function and its regularity properties. The Pontryagin maximum principle is proved and one shows that under some circumstances the value function is the solution of a Hamilton-Jacobi equation.

The last chapter is devoted to control problems with exit time. The structure of the chapter and the range of results proved are close to those from Chapter 6. Supplimentary difficulties appear due to the fact that the time horizon is no longer finite.

Finally, let us mention that the book contains also appendices on the Legendre transform, ordinary differential equations, set-valued analysis and BV functions.

We think that the authors attained their proposed goal. The book is well written, can be easily read even by nonspecialists in the field and, more than that, it constitutes not only an instructive, but also an agreable lecture. We warmly recommend it to the specialists in nonlinear analysis and (or) optimal control theory who may find collected here classical and recent results in their field of interest, as well as to those who like mathematical analysis and to the graduates who want to specialize in this field.

Mihai Pascu

W.O. Amrein, A.M. Hinz and D.B. Pearson (Eds.), *Sturm-Liouville Theory – Past and Present*, Birkhäuser, Basel-Boston-Berlin, 2005, XX+335 pp., ISBN 3-7643-7066-1

Sturm-Liouville equations play a very important role in the theory of (ordinary and partial) differential equations and, consequently, in physics and engineering: in the last paper from the book under review there are described more than 50 examples of Sturm-Liouville equations. Therefore, since the publication of Sturm's pioneering works in the 1830's, Sturm-Liouville theory developed continuously and at present is a very dynamic field. Among those who had crucial contributions to this theory in the first 50 years of 20th century we should mention H. Weyl and E.C. Titchmarsh.

This book is a collection of papers, most of them based on the lectures given by the authors at a Sturm colloquium and workshop held from 15 to 19 September 2003 at the University of Geneva. Some of the papers describe the evolution of the Sturm-Liouville theory since its foundation until now. Other papers present results obtained during recent years. We should mention that almost all the authors have important contributions to the development of the Sturm-Liouville theory in the last decades.

The list of the papers contained in the book gives a good image on the variety of the subjects addressed : D. Hinton, *Sturm's* 1836 *oscillation results. Evolution of the theory*; B. Simon, *Sturm oscillation and comparison theorems*, W.N. Everitt, *Charles Sturm and the development of Sturm-Liouville theory in the years* 1900 *to* 1950; J. Weidmann, *Spectral theory of Sturm-Liouville operators. Approximation by regular problems*; Y. Last, *Spectral theory of Sturm-Liouville operators on infinite intervals. A review of recent developments*; D. Gilbert, *Asymptotic methods in the spectral analysis of Sturm-Liouville operators*; Ch. Bennewitz and W.N. Everitt, *The Titchmarsh-Weyl eigenfunction expansion theorem for Sturm-Liouville differential equations*; V.A. Galaktionov and P.J. Harwin, *Sturm's theorems on zero sets in nonlinear parabolic equations*; C.-N. Chen, *A survey of nonlinear Sturm-Liouville equations*; R. del Rio, *Boundary conditions and spectra of Sturm-Liouville operators*; M.M. Malamud, *Uniqueness of the matrix Sturm-Louville equations given a part of the monodromy matrix, and Borg type results*; W.N. Everitt, *A catalogue of Sturm-Liouville differential equations.*

We think that the book is more than a collection of papers presented at a conference. The papers are carefully written, each of them covers fields of interest not only from the historical point of view, but also for the current research. Therefore, the book can be interesting for a large variety of readers: specialists in differential equations and operator theory, graduate students, scientists working in domains where Sturm-Liouville theory is used.

Mihai Pascu

Jean-Philippe Anker and Bent Ørsted (Eds.), *Lie Theory Harmonic Analysis on Symmetric Spaces – General Plancherel Theorems*, Progress in Mathematics, Vol. 230, Birkhäuser, Boston-Basel-Berlin, 2005, VI+175 pp., ISBN 0-8176-3777-X

The book under review is the third part of the series *Lie Theory* edited by Jean-Philippe Anker and Bent Ørsted. The previous two volumes of this series have the titles *Lie algebras and representations* (2004) and *Unitary Representations and Compactifications of Symmetric Spaces* (2005).

The present volume consists of three articles written by Erik van den Ban, Henrik Schlichtkrull, and Patrick Delorme, respectively. The background of all three articles is the relationship between harmonic analysis on homogeneous spaces and representation theory. Specifically, let G be a real reductive Lie group, $\sigma\colon G \to G$ an involutive automorphism, and H an open subgroup of the fixed-point group of σ. Much of harmonic analysis on the homogeneous space G/H concerns the following problems:

(a) find the decomposition of the regular representation of G in $L^2(G/H)$ as a direct integral of irreducible unitary representations;

(b) find simultaneous spectral decompositions for the G-invariant differential operators on G/H.

The contribution of E. van den Ban in the book under review has the title *The Plancherel theorem for a reductive symmetric space* and provides a self-contained description of the solution to these problems as provided by himself and H. Schlichtkrull in their joint papers published in Inventiones Mathematicae **161** (2005), *no.* 3. The contribution of H. Schlichtkrull concerns a closely related topic, namely, *The Paley-Wiener theorem for a reductive symmetric space* and aims at introducing the reader to the results obtained by himself and E. van den Ban and published in Annals of Mathematics **164** (2006), *no.* 3. On the other hand, the article authored by P. Delorme, *The Plancherel formula on reductive symmetric spaces from the point of view of the Schwartz space*, provides an exposition of the solution that the author and his collaborators J. Carmona, E. van den Ban, and J.L. Brylinski gave to problems (a) and (b) above. This approach was described in a series of papers including for instance the one by P. Delorme published in Annals of Mathematics **147** (1998), *no.* 2.

We should to mention that all contributors to this volume certainly did every effort to make their papers self-contained and to highlight the central ideas in the corresponding expositions. And there is no doubt that the editors of this book managed to publish a collection of articles that will prove to be of a particular importance for the mathematicians interested in harmonic analysis and representation theory of Lie groups, as well as for the students who wish to work in these central areas of contemporary mathematics.

Daniel Beltiţă

Feodor Bogomolov and Yuri Tschinkel (Eds.), *Geometric Methods in Algebra and Number Theory*, Progress in Mathematics, Vol. 235, Birkhäuser, Boston, 2005, VIII+362 pp., ISBN 0-8176-4349-4

The volume offers a representative sample of actual problems and recent results in algebraic and arithmetic geometry. Most results discussed in this volume have been presented at a conference held in Miami, December 2003.

The main themes covered by the articles of this volume and their authors are as follows:

• algebraic curves: unramified covers (Bogomolov-Tschinkel), Jacobians (Zarhin), moduli spaces (Hassett), non-Abelian Hodge theory (Hausel);

• algebraic surfaces: finite group theoretic methods for constructing interesting surfaces (Bauer-Catanese-Grünewald), rational points on cubic surfaces (Swinnerton-Dyer);

• representation-theoretic and combinatorial aspects of higher dimensional geometry (Concini-Procesi, Tamvakis);

• special points and loci on Shimura varieties (Chai, Pink);

• invariants of higher dimensional singular varieties (Budur);

• motivic setup: limit Hodge structures (Spitzweck), p-adic integrals (Cluckers-Loeser);

• function field extensions with prescribed invariants (Ellenberg-Venkatesh);

• algebraic dynamical systems and Arakelov theory (Pineiro-Szpiro-Tucker).

Şerban A. Basarab

Dorin Bucur and Giuseppe Buttazzo, *Variational Methods in Shape Optimization Problems*, Progress in Nonlinear Differential Equations and Their Applications, Vol. 65, Birkhäuser, Boston, 2005, VIII+216 pp., ISBN 0-8176-4359-1

The book under review is a collection of lecture notes from two courses given in the academic year 2000–2001 by the authors at Dipartimento di Matematica dell'Università di Pisa and Scuola Normale Superiore di Pisa, respectively.

In the last years, the captivating field of shape optimization problems has attracted the interest of a huge number of researchers, both in mathematics, as well as in the engineering sciences.

The book, consisting of seven chapters, opens with the general formulation of a shape optimization problem. Roughly speaking, a shape optimization problem is a minimization problem where the unknown variable runs over a class of domains. Such a problem can be written as

$$\min\{F(A) \,:\, A \in \mathcal{A}\},$$

where $\mathcal{A}$ is the class of admissible domains and F is the cost functional, to be minimized over $\mathcal{A}$. The intriguing feature is that the competing objects are *shapes*, i.e., domains in $\mathbb{R}^N$, instead of functions, as is usual in problems of the calculus of variations. Such a constraint often leads to a lack of existence of a solution and to the introduction of suitable *relaxed* formulations of the problem. Also, the first chapter contains some relevant examples of shape optimization problems, such as the isoperimetric problem, the Newton problem of optimal aerodynamical profiles or the optimal distribution of two different media in a given region.

In Chapter 2 the authors consider an important case where the additional constraint of convexity is assumed on the competing domains. This geometrical constraint is usually strong enough to provide the extra compactness necessary to guarantee the existence of an optimal solution.

Many shape optimization problems can be seen in the larger frame work of optimal control problems. An admissible shape plays the role of admissible control in many shape optimization problems. This point along with the relaxation theory is explained in Chapter 3.

Chapter 4 is devoted to shape optimization problems associated with elliptic operators of Dirichlet-Laplacian type. Examples of non-existence and relaxation in optimal shape design are provided and necessary optimality conditions are obtained.

Chapter 5 presents some particular cases, for which, due to geometrical constraints and the monotonicity of the cost functional, classical (unrelaxed) solutions exist.

In Chapter 6 the case of cost functionals depending on the eigenvalues of an elliptic operator with Dirichlet conditions on the free boundary is addressed. Finnally, shape optimization problems with Neumann conditions on the free boundary are addressed in Chapter 7.

The bibliography covers about 200 references of this rapidly developing subject.

The book is written in a very concise and rigorous manner and the style is quite captivating. It covers all the important topics in basic shape optimization theory while many advanced problems of engineering interest are also addressed. Hence, the book proves to be an excellent text for graduate students as well as for researchers in mathematics, engineering, mechanics or physics.

Claudia Timofte

V. Capasso and D. Backstein, *An Introduction to Continuous-Time Stochastic Processes*, Modeling and Simulation in Science, Engineering and Technology, Birkhäuser, Basel-Boston-Berlin, 2005, XI+343 pp., ISBN 0-8176-3234-4

Stochastic processes are more and more used for modeling phenomena of the real life. Therefore scientists working in various fields may be interested in this subject. The aim of this book is to provide them a self-contained and rapid introduction to the theory of continuous-time stochastic processes and to some of its applications. The authors insist on the presentation of the concepts and of the main theorems of the theory. Thus proofs are only given when one considers that they can help the reader to better understand the definitions and theorems or that they may justify methods towards the applications. If a proof of an important result is omitted, then references to the mathematical literature are provided.

The book has three parts. The first part contains four chapters and is devoted to the foundations of the stochastic processes and stochastic calculus. A concise presentation of the fundamentals of the probability theory is given in the first chapter. The main stochastic processes (Gaussian, Markov, martingales, Wiener, Poisson and Lévy) are studied in the second chapter. The Itô integral and the stochastic differential are introduced in the third chapter. The subject of the fourth chapter is stochastic differential equations. It contains important results such as the theorem on the existence and uniqueness of the solutions of the Cauchy problem, Girsanov theorem, the theorem on Kolmogorov equation, Feynman-Kac formula, theorems on the stability of stochastic differential equations.

The second part of the book contains two chapters dedicated to the applications of stochastic processes to finance and insurance (Chapter 5) and to medicine and biology (Chapter 6). Here one can see how to apply the concepts and results from the first part for the study of models which arise in these two areas. Arbitrage free markets, Black-Scholes model, insurance risk and interest rates (in Chapter 5) and population dynamics and Stein's model of neural activity (in Chapter 6) are discussed.

Each chapter contains a section with exercises and additions where further developments are suggested.

The book ends with four appendices: measure and integration, convergence of probability measures on metric spaces, maximum principles of one dimensional elliptic operators and parabolic operators (with one spatial variable), and stability of ordinary differential equations.

We especially recommend this book to professionals from business, biology and medicine who are working with models based on stochastic processes and wish to have a deeper insight into the theory of these processes. It can also be used as a guide by a graduate who wants to specialize in the theory of stochastic processes and of their applications.

Mihai Pascu

Guy David, *Singular Sets of Minimizers for the Mumford-Shah Functional*, Progress in Mathematics, Vol. 233, Birkhäuser Verlag, Basel-Boston-Berlin, 2005, XIV+581 pp., ISBN 3-7643-7182-X; 13: 978-7643-7182-1

The Mumford-Shah functional was introduced in the '80s and is a basic tool for describing various phenomena arising in image segmentation. The main facts can be briefly described as follows. Consider an "image" $g \in L^\infty(\Omega)$, where $\Omega \subset \mathbb{R}^N$ ($N = 2$ in applications) is a simply connected domain. The key idea of the image segmentation is to replace g with a "simpler" image u which captures "the main features" of g. In a first approximation, a good candidate for u is obtained by studying the energy functional $E(u) = \int_\Omega |u-g|^2 \mathrm{d}x$. A more elaborate approach is related to the behaviour of the functional

$$J(u,K) = a\int_\Omega |u-g|^2\mathrm{d}x + b\mathcal{H}^{n-1}(K) + c\int_{\Omega\setminus K} |\nabla u|^2 \mathrm{d}x\,,$$

where a, b and c are positive constants, K is a closed subset of Ω with finite Hausdorff measure, and $u \in H^1(\Omega \setminus K)$. The first part of J is the "fidelity" and expresses the distance between u and g in the L^2-norm. The second term is the $(n-1)$-Hausdorff measure of the singular set K of u (the function u is allowed to jump across K whereas it is assumed to be smooth outside K), while the last summand in the expression of J measures the smoothness of u. The author points out four types of global minimizers of the functional J: (i) constant functions; (ii) functions attaining two values separated by a line; (iii) functions attaining three values separated by a propeller (a union of three half-lines emanating from the same point and making mutually the angle 120 degrees); (iv) "crack-tip" functions, which are special non-constant solutions with a singular half-line.

The monograph consist of nine chapters and develops the basic properties of the Mumford-Shah functional, in connection with many recent results in this field. The functional is introduced in Part A, where there are discussed both the origins of the problem and the Mumford-Shah conjecture in dimension 2, which claims that if u minimizes J, then the singular set K is a finite union of C^1 arcs. The main properties of Sobolev spaces are recalled in the next chapter. Part C is devoted to some refined regularity results of minimizers, such as Carleson measure estimates or concentration properties. Part D includes several sharp recent results. One of the main theorems of this chapter establishes that limits of minimizers or almost minimizers are still minimizers or almost minimizers. Part E deals with the C^1-regularity almost everywhere of almost minimizers and the proofs rely on adequate estimates of the renormalized energy $r^{-1}\int_{B(x,r)\setminus K}|\nabla u|^2 \mathrm{d}x$. Part F contains further properties of global minimizers in the plane, including the blow-up limits of Mumford-Shah minimizers. Additional properties of almost minimizers and regularity results are developed in the next chapter. Part H contains corresponding results in higher dimensions, while the last chapter is devoted to a description of the regularity of K near the boundary. In the particular (and most important) case $N = 2$ it is argued that, near $\partial\Omega$, K is a finite union of C^1 curves that meet $\partial\Omega$ orthogonally.

This book is a good reference for researchers and PhD students in Applied Mathematics. All people interested in modern and powerful directions of the Calculus of Variations may find a good source in the present monograph. The book under review has been awarded the Ferran Sunyer i Balaguer 2004 Prize.

Vicenţiu Rădulescu

Richard H. Enns, *Computer Algebra Recipes for Mathematical Physics*, Birkhäuser, Boston, 2005, XIV+390 pp., ISBN 0-8176-3223-9

The advent of powerful computers and sophisticated symbolic computation packages has deeply modified the way we do research. To a lesser extent and somewhat later on, these changes have also influenced the way sciences are taught. New pedagogical approaches are possible in environments incorporating advanced technology. Traditional textbooks are either replaced or complemented by texts conceived having in view the latest developments.

The present book is a companion to the most successful mathematical physics texts in USA: G.B. Arfken and H.J. Weber, *Mathematical Methods for Physicists*, 5th Ed., Academic Press, New York, 2000; M.L. Boas, *Mathematical Methods in the Physical Sciences*, 2nd Ed., Wiley, New York, 1983; J. Mathews and R.L. Walker, *Mathematical Methods of Physics*, 2nd Ed., Addison-Wesley, New York, 1971. Its contents closely follows the American curriculum for upper undergraduates physics and engineering studies. The book is conceived as a guide to problem-solving in mathematical physics with the help of the computer algebra system MAPLE. Besides being able to perform numerical computations with high precision, MAPLE has capabilities to carry out symbolic integration (i.e., finding primitives), to solve ordinary and partial differential equations by means of various techniques, to simplify and transform expressions involving polynomials, scalar and vector fields in curvilinear coordinates, complex variables, matrices, formal or Laurent series. Additionally, options for various graphical representations of these objects are available.

The reader is supposed to have acquired a certain knowledge of physics before turning to the present book as a first entry-point in the fascinating world of computer-aided problem solving. He/she will find practically no explanations on the physical phenomenon, only detailed descriptions of the MAPLE commands used to set up the mathematical model and then to answer the questions asked for. The relevant commands, introduced on a need-to-know basis, are organised in 230 carefully annotated worksheets. The comments touches upon the underlying mathematics and physics involved in the solution of the problems, but mostly describe the combination of commands which accomplish the task within version 9.5 of MAPLE. The same objective can be achieved by using a different code, another version of MAPLE, or any other computer algebra system. The migration should require a minimum of time, provided the source code and the target environment are well understood.

The material is presented following an amusing analogy with a cook-book, the "recipes" presented here implementing methods to answer important questions and to learn helpful techniques of mathematical physics. Each recipe starts with the analytic formulation of a problem, gives analytic or numerical solution for it, and, whenever appropriate, also provides a graphical visualization of the answer. Various techniques, ranging from contour and vector field plots to static 2-dimensional pictures to animated 3-dimensional graphs, aim to give readers a strong grasp of the physical phenomenon modeled. All recipes are included on an accompanying CD. Additional problems are proposed at the end of each chapter. Complete solutions are found on the CD-ROM. Many more problems can be generated and solved with the same accuracy and easiness by altering the parameter values or initial conditions. This feature stimulates experimentations with various aspects of physical phenomena, resulting in accrued insight.

The book is recommended both for instructors (who can use it in the classroom or as a reference) and students (for self-study or as a text for an online course) in physics and engineering. Once again, this is not a replacement, but a useful extension of classical textbooks in mathematical physics.

Mihai Cipu

Ernst Kunz, *Introduction to Plane Algebraic Curves*, Birkhäuser, Boston-Basel-Berlin, 2005, XII+293 pp., ISBN 0-8176-4381-8; 978-0-8176-4381-2

This book illustrates the pedagogical idea of the author, that the best way to learn commutative algebra is by simultaneously studying applications in algebraic geometry. In the present book, the author studies the local theory of plane algebraic curves, giving in this way an introduction to commutative algebra via plane algebraic curves. According to this idea, the book differs consistently from other texts on plane algebraic curves, by focusing on their algebraic aspects and only casually refering to their topological or analytic features.

The main objects of study of the book are plane algebraic curves over an algebraically closed field. Part I covers facts about algebraic curves: affine and projective algebraic curves, coordinate ring of an algebraic curve, rational functions, intersection multiplicity on intersection cycle of two curves, regular and singular points, tangents, rational maps, polars and hessians, elliptic curves, residue calculus with applications, the Riemann-Roch theorem, genus of an algebraic curve, canonical divisor class, branches, conductor and value semigroup of a curve singularity. Part II contains the algebraic background needed.

One of the most interesting features of the book is the presentation of the intersection theory of plane curves via filtered algebras, their graded rings and Rees algebras and by applying the algebraic residue calculus. Also, a new version of F.K. Schmidt's classical proof for the Riemann-Roch theorem for plane curves is presented. A multitude of very well chosen examples and applications are spread throughout the book.

This is a wonderfully written book. The pedagogical skills of the author, already well-known from other textbooks, are coming out more than ever. The book is extremely useful for students, researchers in commutative algebra or algebraic geometry, for all those who want to get acquainted with a very interesting part of mathematics, written in a delightful way.

Cristodor Ionescu

Petr P. Kulish, Nenad Manojlovic and Henning Samtleben (Eds.), *Infinite Dimensional Algebras and Quantum Integrable Systems*, Progress in Mathematics, Vol. 237, Birkhäuser Verlag, Basel-Boston-Berlin, 2005, VIII+263 pp., ISBN-10: 3-7643-7215-X; 13: 978-3-7643-7215-6

The applications of infinite-dimensional Lie groups and algebras in various areas of mathematical physics have by now a long and well-established tradition. And a crucial role in the developments along this line of research was played by Victor Kac, one of the contributors to the volume under review. This book actually collects the expanded versions of the invited lectures at a satellite workshop of the 14th International Congress of Mathematical Physics. The workshop had the same title as the book under review and was held in July 2003 at the University of Algrave in Faro, Portugal. The volume consists of the following contributions: Edward Frenkel, *Gaudin model and opers*; O. Castro-Alvaredo and A. Fring, *Integrable models with unstable particles*; V.G. Kac and M. Wakimoto, *Quantum reduction in the twisted case*; A. Gerasimov, S. Kharchev and D. Lebedev, *Representation theory and quantum integrability*; H.E. Boos, V.E. Korepin and F.A. Smirnov, *Connecting lattice and relativistic models via conformal field theory*; K. Takasaki, *Elliptic spectral parameter and infinite-dimensional Grassmann variety*; T. Takebe, *Trigonometric degeneration and orbifold Wess-Zumino-Witten model*, II; L.A. Takhtajan and L.-P. Teo, *Weil-Petersson geometry of the universal Teichmüller space*; V. Tarasov, *Duality for Knizhinik-Zamolodchikov and dynamical equations, and hypergeometric integrals*.

We emphasize that all the papers in this volume are authored by leading experts in the field and are highly reliable surveys of recent developments. Thus, E. Frenkel's paper is an account of researches originating in his collaboration with B. Feigin and N. Reshetkin on the Gaudin Hamiltonians associated with a complex semisimple Lie algebra. The paper by O. Castro-Alvaredo and A. Fring describes a Lie algebraic picture of integrable quantum field theories in $1+1$ space-time dimensions with an infinite spectrum of unstable particles.

In the contribution authored by V.G. Kac and M. Wakimoto one can find a unified representation theory of all twisted superconformal algebras; in addition, the authors announce a new theory of characters for a class of vertex algebras introduced here. The object of the survey by A. Gerasimov, S. Kharchev and D. Lebedev is to describe a number of constructions in representation theory of classical and quantum groups, and to use these constructions in order to set up a direct relationship between representation theory and the quantum inverse scattering method. The paper by H.E. Boos, V.E. Korepin and F.A. Smirnov is devoted to a relativistic model of conformal field theory which is quantum group invariant, a special attention being paid to the infrared and thermodynamic limits.

The relationship between recently discovered soliton equations and Grassmann manifolds is treated by K. Takasaki in a very lucid manner, with a special emphasis on the holomorphic bundles that naturally live on the manifolds of the aforementioned type. The paper by T. Takebe is the sequel to one published in the *Proceedings of the* 6*th International Workshop on Conformal and Integrable Models.* Its aim is to point out that the trigonometric Wess-Zumino-Witten model is indeed the degenerate twisted Wess-Zumino-Witten model on elliptic curves as defined in a previous paper by G. Kuroki and the present author.

The contribution by L.A. Takhtajan and L.-P. Teo is a brief account on recent advances in the geometry of the universal Teichmüller space $T(1)$, which is a complex Banach manifold and contains the Teichmüller spaces of Riemann surfaces as complex submanifolds. Finally, the paper by V. Tarasov describes the hypergeometric solutions to the Knizhinik-Zamolodchikov equations, and the duality referred to in the title of the paper leads to various identities involving hypergeometric and q-hypergeometric dimensions.

It is clear from this sketchy description of the contents that this volume gathers a wide range of ideas and techniques belonging to the boundary between Lie theory and mathematical physics. We can safely say that the book will not fail to appeal to the researchers in such areas as infinite-dimensional Lie theory, algebraic geometry, integrable systems, or conformal field theory.

Daniel Beltiţă

A.J. Kurdila and M. Zabarankin, *Convex Functional Analysis*, Systems Control: Foundation & Applications, Birkhäuser, Basel-Boston-Berlin, 2005, XIV+228 pp., ISBN 3-7643-2198-9

This is the first from a series of two volumes devoted to the presentation of the basic concepts and results of convex analysis and its applications in mechanics and control theory. The volume under review, is devoted to the fundamentals of convex functional analysis.

The monograph contains seven chapters. The first five chapters treat general topics in analysis and functional analysis: topology and Lebesgue integrability (Chapter 1), linear functionals and linear operators (Chapter 2), L^p spaces, Sobolev spaces, Banach space valued functions (Chapter 3), Gateaux and Fréchet differentiability (Chapter 4) and minimization of functionals (Chapter 5). The topics treated in the last two chapters are more specific to convex functional analysis: convex functionals, convex programming and ordered vector spaces (Chapter 6), lower semicontinuous functionals and convexity, generalized Weierstrass theorem (Chapter 7).

The authors tried, successfully we believe, to make the presentation accessible to those who are not familiarized with graduate mathematics. To this end, they included a lot of examples which can help the reader to better undestand the definitions and theorems contained in the text.

Mihai Pascu

Ivo Nowak, *Relaxation and Decomposition Methods for Mixed Integer Nonlinear Programming*, International Series of Numerical Mathematics, Vol. 152, Birkhäuser, Basel, 2005, XVI+213 pp., ISBN 3-7643-7238-9

The study of mixed integer nonlinear problems (MINLP) is a very active research area because optimization problems containing continuous as well as discrete variables arise in engineering, communications, finance, and many other fields. While the algorithms for mixed integer linear problems are quite efficient, the most successful ones dealing with models with millions of variables and constraints, the performances dramatically downgrade in the presence of nonlinear constraints. In the last ten years or so, many ideas evolved in MINLP and a lot of work has been done to compare their efficiency.

In this book, a generic branch-cut-and-price framework for solving nonconvex structured MINLPs is proposed. The idea is to extend the underlying concept in most of the methods addressing mixed integer linear problems. In the process of generalization, basic optimization tools are adapted. The first part of the book, consisting of nine chapters, contains the theoretical background, which includes block-separable reformulations of MINLP, convex, semidefinite programming and Lagrangian relaxations, decomposition methods and global optimality criteria. Some of the results are quite new and are presented here for the first time. The theoretical and algorithmic advances allow the author to develop an object oriented library, called LaGO (Lagrangian Global Optimization), for solving nonconvex mixed-integer all-quadratic programs.

The second part deals with the algorithmic aspects of the area. One short chapter is devoted to an overview of the existing global optimization methods. Next, one describes deformation, rounding, partitioning and Lagrangian heuristics, all aiming at transforming a difficult optimization problem into a relaxed problem that is easier to solve. Thus, one generates a sequence of relaxed problems whose solutions converge towards the solution of the original problem. The last two chapters contain a presentation of the branch-and-cut-price algorithms for general MINLPs and an overview of the library LaGO, respectively.

The book is very well written, the material is well organized, the ideas are succinctly and carefully formulated. Results of numerical experiments with several packages developed to solve MINLPs are reported. The computing environments and the MINLP instances used in comparisons are carefully described, so that the conclusions of experiments are well-founded. An up-to-date bibliography, with more than 200 entries, many of which are addresses of sites of interest for people working in this area, is another strong point of the book. I am convinced that this monograph will find a wide audience: it is suitable for teaching at the graduate level and may as well serve as a reference text for research in the area of MINLP.

Mihai Cipu

Piotr Pragacz (Ed.), *Topics in Cohomological Studies of Algebraic Varieties*, Impanga Lecture Notes, Birkhäuser, Basel, 2005, XXVIII+297 pp., ISBN 3-7643-7214-1

The volume under review is a collection of lecture notes given in Warsaw by several experts in algebraic geometry. As the title shows, they focus primarily on cohomological aspects of the study of projective varieties.

The first lecture, by P. Pragacz, is a captivating resumé of the life and work of A. Grothendieck – unquestionably one of the masters of modern mathematics. The present volume is dedicated to him.

P. Aluffi lectures on characteristic classes of singular varieties. He discusses Chern-Schwartz-MacPherson and Chern-Fulton classes, and gives concrete ways of computing them. One of the goals of the lectures is to explain how the computation is performed with help of `Macaulay2`, via the script `CSM.m2`.

M. Brion discusses the geometry of flag varieties. In the first part of these self-contained notes, the basic definitions are recalled: Grassmannians, flag varieties, Schubert varieties. Then the author advances to modern problems: singularities of Schubert varieties and their resolutions, cohomology of line bundles. The Grothendieck ring of a flag variety occupies is paid special attention.

A.S. Buch lectures on Grothendieck polynomials and relations with the Grothendieck ring of Grassmannians.

The next notes, of H. Duan, are on Morse functions with applications to flag and Schubert varieties. In particular, it is shown how Bott-Samelson type functions appear in connection with resolutions of Schubert varieties.

A.U.O. Kisisel gives a self-contained introduction to integrable systems, KP and KdV hierarchies, and relations with the Gromov-Witten theory. The final part briefly explains Kontsevich's solution to the Witten conjecture, and opens the door for further developments.

P. Pragacz' second notes study Schubert calculus, indicating how to multiply Schubert classes of generalized flag varieties.

J. Schürmann discusses characteristic classes associated with constructible functions, used on singular varieties. The Verdier-Riemann-Roch theorem is presented, too.

M. Szyjewski provides an introduction to algebraic K-theory of schemes, illustrated with several concrete examples. The case of quadrics, for instance, is worked out in detail.

H. Tamvakis proves that the three-point genus zero Gromov-Witten invariants are the same as the classical triple intersection numbers on homogeneous spaces of the same Lie type, and presents the quantum Pieri rule for Lagrangian Grassmannian and orthogonal Grassmannian.

M. Aprodu

Omri Rand and Vladimir Rovenski, *Analytical Methods in Anisotropic Elasticity with Symbolic Computational Tools*, Birkhäuser, Boston-Basel-Berlin, 2005, XVIII+ 451 pp., ISBN 0-8176-4272-2

This is a most comprehensive textbook on the subject. The authors refresh and modernize classical mathematical methods encountered in the theory of anisotropic elasticity (modeling and analytical solutions) since exact and approximate mathematical solutions are really important for their physical interpretation which provides important insight into the phenomena involved. At the same time, they also prove the potential of modern symbolic computational tools to support highly complex analytical solutions and their contribution to the rigor, analytical uniformity and exactness of the derivation. The book provides an excellent theoretical background for composite materials analysis.

The book is well structured, into 11 chapters, from such basic notions as deformation measures, strain, stress measures, energy theorems, Euler's equations, etc., up to most advanced formulations such as a complete analytical model and solution scheme for an arbitrarily loaded non-homogeneous beam structure of generic anisotropy.

There are two distinct parts. Part I (Chapters 1–4) contains the basics in anisotropic elasticity: fundamentals of anisotropic elasticity and analytical methodologies (Chapter 1), anisotropic materials (Chapter 2), plane deformation analysis (Chapter 3) and solution methodologies (Chapter 4). Part II (Chapters 5–10) is devoted to various beam analyses and contains recent and advanced models developed by the authors: foundation of anisotropic beam analysis (Chapter 5), beams of general anisotropy (Chapter 6), homogeneous, uncoupled monoclinic beams (Chapter 7), non-homogeneous plane and beam analysis (Chapter 8), solid coupled monoclinic beams (Chapter 9) and thin-walled coupled monoclinic beams (Chapter 10). The book also contains a CD with symbolic codes (MAPLE, versions 8 and 9) for the solutions presented, that may be activated to create numerous additional examples containing detailed expressions and graphics. Chapter 11 presents instructions for the symbolic and illustrative programs included in the book.

The book is intended to a broad audience involved in mathematical modeling: graduate students, professors, engineers, applied mathematicians, numerical analysis experts, researchers in continuum mechanics.

Mihaela Mihăilescu-Suliciu

Titu Andreescu and Dorin Andrica, *Complex Numbers from A to ... Z*, Birkhäuser, Boston-Basel-Berlin, 2006, XI+321 pp., ISBN 0-8176-4326-5

The history of complex numbers starts in 18th century with L. Euler who broke the frontiers of "common sense" and introduced the imaginary unit $\mathrm{i} = \sqrt{-1}$ to be solution of the quadratic equation $x^2 + 1 = 0$. The complex numbers are then defined to be the numbers of the form $a + \mathrm{i}b$ with a and b real numbers and the algebraic operations are defined in a natural way.

Passing from algebra through trigonometry into geometry, the complex numbers proved to be very useful in offering insights into problems which apparently had nothing in common with them. And as "imaginary" as they might be, they are a common presence in applied sciences as mechanics, physics or chemistry.

The target audience of the book are maths undergraduates, highschool students and their teachers. The most difficult part of the book can be very well used for training the students who wish to compete at mathematics competitions as Olympiads or the W.L. Putnam Mathematical Competition. The book is self-contained, the only requirement is a working knowledge of real numbers and their properties. As the title suggests, the new book of Andreescu and Andrica offers to the target reader a comprehensive, yet gradual exposition of the field of complex numbers.

The book consists of five chapters and a sixth one with solutions to all the problems left as exercises throughout the book. Each of the first five chapters starts with an extensive theoretical part. Here most of the results are given full proofs and several applications are presented. Then, a list of problems to be solved follows. The first two chapters deal with complex numbers in algebraic and trigonometric form, respectively. The third chapter insists on the connection between complex numbers and geometry. This comes from identifying each point in the plane with a complex number. Then, metric invariants or configurations of points in the Euclidean plane (lines, distances, measure of angles, circles, etc.) have explicit algebraic description. This "dictionary" is then used in Chapter 4 to give new (sometimes simpler) solutions to problems in plane geometry or to study some geometric transformations of the complex plane. As an application of these tools, the proof of Alain Connes for Morley's theorem is given. The fifth chapter, which is also the largest one, consists of Olympiad caliber problems and here several major classes of problems are identified. The book also contains a glossary of terms, a rich list of references, an index of authors of problems and a subject index.

Originated in the vast experience of the authors in preparing students for mathematics competitions, the present book is useful both for beginners and advanced students.

Dumitru I. Stamate

B.S. Mordukhovich, *Variational Analysis and Generalized Differentiation* I, II, Grundlehren der mathematischen Wissenschaften, Vol. 330, Springer, Berlin-Heidelberg-New York, 2006, XXII+580 pp., ISBN 3-540-25437-4 (Vol. I), XXII+610 pp., ISBN 3-540-25438-2 (Vol. II)

Various problems arising in mathematics and applied mathematics can be reduced to the following one: find the extremal points of an appropriate function. A first answer to this problem was given by Fermat in his famous theorem on stationary points. Since then, attempts to obtain more general results which can help in solving larger classes of problems led to a steady development of nonlinear mathematical analysis and, especially, of variational analysis.

This monograph presents some of the last developments in this direction and their applications to optimization problems.

The book is a two-volume one, each one containing four chapters. The first volume presents the general theory. Topics as generalized differentiation of multifunctions and of functions on infinite dimensional Banach spaces, extremal and variational principles and sensitivity (stability) analysis are covered. The second volume is devoted to applications to constrained optimization and equilibria, optimal control of evolution and distributed systems, and mathematical economics.

Each chapter ends with a generous section of comments on the history and motivation of the notions and results which make its object. Comments which prove the author's deep knowledge of the subject can also be found throughout the book.

Mention should be also made of the quite impressive list of references: it contains 1379 items, B.S. Mordukhovich being author or joint author of more than 90 of them.

Let us now shortly describe the content of each chapter.

Chapter 1, *Generalized differentiation in Banach spaces.* A geometric dual space approach to generalized differentiation in a general Banach spaces setting is developed. There are introduced notions which are intensively used later (generalized normals, tangent cones to nonconvex sets, sequential normal compactness of sets and of mappings, coderivatives, generalized Lipschitz-like properties, metric regularity and covering, subdifferentials). The relations between them are put into evidence. Calculus rules for the generalized normals, coderivatives, subdifferentials, to be refined in Chapter 3, are first obtained here.

Chapter 2, *Extremal principle in variational analysis.* Several versions of the extremal principle are defined. Extremal principles can be viewed as a local variational analogue of separation theorem from convex analysis. They will be intensively used in the next chapters.

One proves that the exact extremal principle holds in finite dimensional spaces. Characterizations of Asplund spaces (Banach spaces which have the property that every convex continuous function defined on an open convex subset U is Fréchet differentiable on a dense subset of U) in terms of the extremal principles are obtained. A basic tool in the proofs is the separable reduction technique. The author discusses also the relations with variational principles. The extremal and variational principles are then used to obtain better representations of the generalized constructions from the previous chapter in the case of Asplund spaces.

Chapter 3, *Full calculus in Asplund spaces.* As we already mentioned, the calculus rules for generalized normals, coderivatives, subdifferentials and the sequential normal compactness calculus for sets and mappings studied in Chapter 1, are refined in the case of Asplund spaces. It appears that Asplund spaces are the natural spaces on which a comprehensive nonsmooth, nonconvex calculus of the generalized differential constructions can be developed.

Chapter 4, *Characterization of well-posedeness and sensitivity analysis.* First of all, neighborhood and point based characterizations of Lipschitz-like property, metric regularity and covering properties for multifunctions in terms of their coderivatives are obtained in Asplund spaces. These characterizations and the calculus rules of generalized differentiation obtained in the previous chapters are then applied to the sensitivity analysis of constraint systems and generalized equations.

The constraint systems (depending on a parameter x) are described by multifunctions $F : X \to Y$ of the form $F(x) = \{y \in Y;\ g(x, y) \in \Theta,\ (x, y) \in \Omega\}$, where $g : X \times Y \to Z$ is a single valued mapping, X, Y, Z are Banach spaces, Θ is a subset of Z and Ω is a subset of $X \times Y$. Such systems can be viewed as generalizations of feasible solution sets to perturbed problems in nonlinear programming.

The generalized equations are of the form $0 \in f(y) + Q(y)$, where f is a single valued mapping and Q is a multifunction between Banach spaces. They are generalizations of classical variational inequalities.

In this chapter one studies also the behavior of metric regularity under perturbations.

The second volume starts with Chapter 5, *Constrained optimization and equilibria.* The tools developed in the first volume – generalized differentiation and extremal/variational principles – are used to obtain necessary conditions for the extremal points for problems of mathematical programming with geometric, operator or functional constraints, as well as for mathematical programs with equilibrium constraints. Multiobjective constrained optimization problems, i.e., problems where the objective function takes values in a Banach space and the optimization is conducted by preference relations, are also treated in this chapter. The last section deals with some generalizations of the notions of set extremality and optimal solutions, namely, set subextremality at a linear rate and suboptimality at a linear rate. Necessary and sufficient conditions are obtained in this case.

Chapter 6, *Optimal control of evolution systems in Banach spaces.* The Bolza problem for differential evolution inclusions is studied in the first section of the chapter. Necessary Euler-Lagrange type optimality conditions are first obtained for discrete-time evolution problems. The necessity of similar conditions for continuous-time relaxed problems is next proved by an approximation procedure. In the second section necessary optimality conditions of Weierstrass-Pontryagin type for differential inclusions without relaxation (nonconvex differential inclusions) are given. In the third section the specific features of optimal control problems governed by ordinary differential equations with smooth dynamics are studied. A special attention is paied to transversality conditions. The last section tries to answer the following question: in which cases is it possible to obtain an analogue of the Pontryagin maximum principle for optimal control problems governed by nonconvex finite-difference systems that approximate a continuous-time problem? The answer is not so simple. The examples and results which are given show the complexity of the problem.

Chapter 7, *Optimal control of distributed systems.* Optimal control problems for differential-algebraic systems with time delays, for semilinear hyperbolic equations with Neumann boundary conditions and pointwise constraints on control function and on state function, for linear hyperbolic equations with Dirichlet boundary conditions and pointwise constraints on control function and on state function, and for parabolic systems with Dirichley boundary conditions, pointwise constraints on state and uncertain perturbations are considered in this chapter. Most of the material presented here is based on results obtained by the author and his collaborators.

Chapter 8, *Applications to economics.* This chapter is essentially devoted to the study of models of welfare economics in classical and advanced frameworks. The extremal principle of variational analysis is used to prove versions of the extended second welfare theorem for Pareto optimal allocations with marginal prices.

To conclude the author succeeded to place at our disposal a comprehensive and deep insight into a fruitful and continuously developing area in mathematics. He presented the theory of modern variational analysis and generalized differentiation in its full generality. This allowed to cover a large range of applications. We appreciate that the book synthetize a life time work of an important mathematician, and warmly recommend it to specialists in mathematical analysis, differential equations or mathematical economics.

Mihai Pascu

Ivan Singer, *Duality for Nonconvex Approximation and Optimization*, CMS Books in Mathematics, Vol. 34, XVIII+355 pp., Springer, New York, 2006

This is a nice addition to the literature on nonconvex optimization in locally convex spaces, devoted primarily to nonconvex duality. Most of the material appears for the first time in book form and examples are abundant. In fact, the book deals with worst approximation, duality for quasi-convex supremization, optimal solution for quasi-convex maximization, reverse convex best approximation, unperturbational duality for reverse convex infimization, optimal solutions for reverse convex infimization, duality for optimization problems involving differences of convex functions and, finally, duality for optimization in the framework of abstract convexity. The concluding chapter contains some comments, bibliographical references and additional results to each of the preceding chapters. The style is friendly. I strongly recommend this book to graduate students studying nonconvex optimization theory.

Constantin P. Niculescu