LOCAL TRAJECTORIES ON KLEIN SURFACES

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The main purpose of this paper is to give a way to define and investigate the local trajectory structure of a N-quadratic differential on a Klein surface.

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The local trajectory structure can be studied on a Klein surface (X, A) itself, but in this paper this is done in terms of the lifts to the double covering.

Let (X, A) be a Klein surface, where X is a surface and A is a maximal dianalytic atlas on X, such that A does not contain any analytic subatlas. There exists the orientable two-sheeted covering (O_2, k) of X, where k is an antianalytic involution of O_2, without fixed points. Let $\widehat{X}$ be the universal covering of X, G the group of covering transformations of X and G_1 the subgroup of G consisting of all conformal covering transformations. Then there exist the corresponding canonical covering projections $p : \widehat{X} \to O_2$, $q : O_2 \to X$ and $\pi : \widehat{X} \to X$. Also, $\widehat{X}/G_1$ is canonically identified with O_2, $\widehat{X}/G$ is canonically identified with X, and X will be also identified with the orbit space O_2/H, where H is the group consisting of the identity of O_2 and k, under the usual composition of functions (see [3]).

For every $T \in G_1$, define

$$h_{T,U} = \left(q_{|U} \circ p_{|T(\widehat{U})}\right)^{-1} : \widetilde{U} \to T(\widehat{U}),$$

where $\widehat{U}$ is a fixed component of $p^{-1}(U)$, so that $p_{|\widehat{U}} : \widehat{U} \to U$ and $q_{|U} : U \to \widetilde{U}$ are homeomorphisms.

For every $S \in G \setminus G_1$, define

$$g_{S,U} = \left(q_{|k(U)} \circ p_{|S(\widehat{U})}\right)^{-1} : \widetilde{U} \to S(\widehat{U}).$$

Therefore, $A = A_1 \cup A_2$, where

$$A_1 = \left\{(\widetilde{U}, h_{T,U}, T(\widehat{U})) \mid\ \widetilde{U} \text{ is a parametric disk on } X \text{ as before, } T \in G_1\right\}$$

and

$$A_2 = \{(\widetilde{U}, g_{S,U}, S(\widehat{U})) \mid \widetilde{U} \text{ is a parametric disk on } X \text{ as before, } S \in G \setminus G_1\},$$

is a dianalytic atlas on X (see [4]).

For convenience of notation, the points and the curves in O_2 and X will be identified with their images in the Euclidean plane, through the corresponding charts.

If $\widetilde{U}$ is a parametric disk on X, then $q^{-1}(\widetilde{U}) = \{U, k(U)\}$ is a pair of k-symmetric disks on O_2. Thus, for the local study of the meromorphic quadratic differentials on O_2, it is natural to consider restrictions $U \cup k(U)$. Since k is an involution without fixed points, one can suppose that $U \cap k(U) = \emptyset$.

Let $Q^2(O_2)$ be the vectorial space of the meromorphic quadratic differentials on (O_2, A_1) and $\overline{Q^2(O_2)}$ the vectorial space of the meromorphic quadratic differentials on (O_2, A_2), that is, the vectorial space of the antimeromorphic quadratic differentials on (O_2, A_1) (see [5]).

Theorem 1. *There is an isomorphism K between $Q^2(O_2)$ and $\overline{Q^2(O_2)}$.*

Proof. Let z be the local parameter on U and w the local parameter on $k(U)$. If φ is not holomorphic, namely, it has at least a pole, then $z \in U$ means z is not a pole of φ. Let $\Phi \in Q^2(O_2)$ with the local representation

$$\Phi_{|U \cup k(U)} = \begin{cases} \varphi(z)\mathrm{d}z^2 & \text{if } z \in U, \\ \widehat{\varphi}(w)\mathrm{d}w^2 & \text{if } w \in k(U), \end{cases}$$

where φ and $\widehat{\varphi}$ are meromorphic functions on U, respectively $k(U)$. Then the symmetry k will induce the isomorphism $K : Q^2(O_2) \to \overline{Q^2(O_2)}$ defined as

$$K(\Phi)_{|U \cup k(U)} = (\Phi \circ k)_{|U \cup k(U)} = \begin{cases} \widehat{\varphi}(k(z))\mathrm{d}k(z)^2 & \text{if } z \in U, \\ \varphi(k(w))\mathrm{d}k(w)^2 & \text{if } w \in k(U). \end{cases}$$

Thus,

$$K(\Phi)_{|U \cup k(U)} = \begin{cases} \widehat{\varphi}(k(z)) \left(\dfrac{\partial k}{\partial \overline{z}}(z) \right)^2 \mathrm{d}\overline{z}^2 & \text{if } z \in U, \\ \varphi(k(w)) \left(\dfrac{\partial k}{\partial \overline{w}}(w) \right)^2 \mathrm{d}\overline{w}^2 & \text{if } w \in k(U), \end{cases}$$

because k is an antianalytic function.

Let $\widetilde{\Phi}$ be an N-meromorphic quadratic differential on X, with $\widetilde{\Phi} \neq 0$, i.e., there is a meromorphic quadratic differential Φ on O_2 such that

$$\widetilde{\Phi}_{|\widetilde{U}} = \Phi_{|U} + K(\Phi)_{|U} = \Phi_{|U} + (\Phi \circ k)_{|U}$$

for every $\widetilde{U}$ a parametric disk on X, where $\widetilde{U} = q(U) = q(k(U))$ (see [4]).

The local representation of $\widetilde{\Phi}$ with respect to a parameter $\widetilde{z} \in \widetilde{U}$, where $q^{-1}(\widetilde{z}) = \{z, k(z)\}$, φ and $\widehat{\varphi}$ are meromorphic functions with respect to z, respectively $k(z)$, is $\widetilde{\Phi}(\widetilde{z}) = \varphi(z)\mathrm{d}z^2 + \widehat{\varphi}(k(z))\mathrm{d}k(z)^2$ (see [5]). $\square$

THEOREM 2. *If $A = \{(\widetilde{U}_i, h_i, V_i) \mid i \in I\}$ is a dianalytic atlas on X, the following assertions are equivalent.*

(1) *There exists an N-meromorphic quadratic differential $\widetilde{\Phi}$ on X.*

(2) *There exists a family of mappings $\widetilde{\varphi}_A = \{\widetilde{\varphi}_i : \widetilde{U}_i \to \widehat{\mathbf{C}} \mid i \in I\}$ such that:*

(2′) *For every $i \in I$, $\widetilde{\varphi}_i \circ h_i^{-1} : h_i(\widetilde{U}_i) \to \widehat{\mathbf{C}}$ is a meromorphic function, where $\widetilde{\varphi}_i \circ h_i^{-1} = \Phi_{|U_i}$ if $h_i \in A_1$, and $\widetilde{\varphi}_i \circ h_i^{-1} = \Phi_{|k(U_i)}$ if $h_i \in A_2$.*

(2″) *If $(\widetilde{U}, h, V)$ and $(\widetilde{U}', h', V')$ are charts from A corresponding to the same point $\widetilde{z}_0 \in X$, with $h(\widetilde{z}_0) = z_0$, respectively $h'(\widetilde{z}_0) = w_0$, and $\widetilde{\Phi}$ is an N-meromorphic quadratic differential on X, $\widetilde{\Phi} \neq 0$, with the local representations $\widetilde{\Phi}(\widetilde{z}) = \varphi(z)\mathrm{d}z^2 + \widehat{\varphi}(k(z))\mathrm{d}k(z)^2$, respectively $\widetilde{\Phi}'(\widetilde{z}) = \varphi'(w)\mathrm{d}w^2 + \widehat{\varphi}'(k(w))\mathrm{d}k(w)^2$ near $\widetilde{z}_0$, if $\widetilde{U}^*$ is a connected component of $\widetilde{U} \cap \widetilde{U}'$, then (see [4])*

$$\begin{cases} \varphi(z) = \varphi'(w)\left(\dfrac{\mathrm{d}w}{\mathrm{d}z}\right)^2 \\ \widehat{\varphi}(k(z)) = \widehat{\varphi}'(k(w))\left(\dfrac{\mathrm{d}k(w)}{\mathrm{d}k(z)}\right)^2 \end{cases}$$

if $h' \circ h^{-1}$ is analytic on $h(\widetilde{U}^*)$, or

$$\begin{cases} \varphi(z) = \widehat{\varphi}'(k(w))\left(\dfrac{\mathrm{d}k(w)}{\mathrm{d}z}\right)^2 \\ \widehat{\varphi}(k(z)) = \varphi'(w)\left(\dfrac{\mathrm{d}w}{\mathrm{d}k(z)}\right)^2 \end{cases}$$

if $h' \circ h^{-1}$ is antianalytic on $h(\widetilde{U}^*)$.

It was shown in [6] that even if the value of the quadratic differential Φ at a point $P \in O_2$ depends on the local parameter near P, it makes sense to define the zeroes and the poles of Φ.

PROPOSITION 1 ([6]). *The order of a critical point of a quadratic differential on O_2 is an analytic invariant, that is, it does not depend on the choice of the local parameter,* [6].

Let $\widetilde{z}_0 \in X$. Then near $\widetilde{z}_0$ we have the representation

$$\widetilde{\Phi}(\widetilde{z}) = \sum_{l=n}^{\infty} a_l (z - z_0)^l \mathrm{d}z^2 + \sum_{l=\widehat{n}}^{\infty} b_l \left[k(z) - k(z_0)\right]^l \mathrm{d}(k(z))^2$$

of $\widetilde{\Phi}$, with $a_n \neq 0$, $b_{\widehat{n}} \neq 0$.

By definition, the order of φ near z_0 is n and we write $\operatorname{ord}(\varphi, z_0; h) = n$ while the order of $\widehat{\varphi}$ near $k(z_0)$ is $\widehat{n}$ and we write $\operatorname{ord}(\widehat{\varphi}, k(z_0); h) = \widehat{n}$.

Theorem 3. *If $(\widetilde{U}, h, V)$ and $(\widetilde{U}', h', V')$ are charts from A corresponding to the same point $\widetilde{z}_0 \in X$, with $h(\widetilde{z}_0) = z_0$, respectively $h'(\widetilde{z}_0) = w_0$, and $\widetilde{\Phi}$ is an N-meromorphic quadratic differential on X, $\widetilde{\Phi} \neq 0$, with the local representations $\widetilde{\Phi}(\widetilde{z}) = \varphi(z)\mathrm{d}z^2 + \widehat{\varphi}(k(z))\mathrm{d}k(z)^2$, respectively $\widetilde{\Phi}'(\widetilde{z}) = \varphi'(w)\mathrm{d}w^2 + \widehat{\varphi}'(k(w))\mathrm{d}k(w)^2$ near $\widetilde{z}_0$, then:*

$$\begin{cases} \operatorname{ord}(\varphi, z_0; h) = \operatorname{ord}(\varphi', w_0; h') \\ \operatorname{ord}(\widehat{\varphi}, k(z_0); h) = \operatorname{ord}(\widehat{\varphi}', k(w_0); h') \end{cases}$$

if $h' \circ h^{-1}$ is analytic, or

$$\begin{cases} \operatorname{ord}(\varphi, z_0; h) = \operatorname{ord}(\widehat{\varphi}', k(w_0); h') \\ \operatorname{ord}(\widehat{\varphi}, k(z_0); h) = \operatorname{ord}(\varphi', w_0; h') \end{cases}$$

if $h' \circ h^{-1}$ is antianalytic.

Proof. The result is a direct consequence of Theorem 2 and Proposition 1. □

For every $\widetilde{z} \in X$ define the set $O(\widetilde{z}) = \{(n, \widehat{n}; h) \mid n, \widehat{n} \in \mathbf{Z},\ (\widetilde{U}, h, V) \in A, \widetilde{z} \in \widetilde{U}\}$ and an equivalence relation "$\sim$" on $O(\widetilde{z})$ by $(n, \widehat{n}; h) \sim (n', \widehat{n'}; h') \Leftrightarrow$ a) $n = n'$, $\widehat{n} = \widehat{n'}$, if $h' \circ h^{-1}$ is an analytic mapping, or b) $n = \widehat{n'}$, $\widehat{n} = n'$, if $h' \circ h^{-1}$ is an antianalytic mapping.

Let $O(\widetilde{z})/\sim$ the set of the corresponding equivalence classes and $[n, \widehat{n}; h] \in O(\widetilde{z})/\sim$ the equivalence class of $(n, \widehat{n}; h) \in O(\widetilde{z})$. By definition,

(e1): $[0, 0; h] = 0$;

(e2): $[n, \widehat{n}; h] \geq 0 \Leftrightarrow n \geq 0,\ \widehat{n} \geq 0$;

(e3): $-\ [n, \widehat{n}; h] = [-n, -\widehat{n}; h]$;

(e4): $[n, \widehat{n}; h] + [n', \widehat{n'}; h] = [n + n', \widehat{n} + \widehat{n'}; h]$;

(e5): $[n, \widehat{n}; h] \geq [n', \widehat{n'}; h] \Leftrightarrow n \geq n',\ \widehat{n} \geq \widehat{n'}$ (see [3]).

The order of an N-meromorphic quadratic differential $\widetilde{\Phi}$ at $\widetilde{z}_0$ will be denoted by $\operatorname{ord}(\widetilde{\Phi}, \widetilde{z}_0)$ and, by definition,

$$\operatorname{ord}(\widetilde{\Phi}, \widetilde{z}_0) = [\operatorname{ord}(\varphi, z_0; h), \operatorname{ord}(\widehat{\varphi}, k(z_0); h); h]\,.$$

Remark 1. It follows from Theorem 3 that the order of an N-meromorphic quadratic differential $\widetilde{\Phi}$ at $\widetilde{z}_0$ is well defined, that is, it does not depend on the choice of the local parameter. Thus, the order of an N-meromorphic quadratic differential $\widetilde{\Phi}$ at $\widetilde{z}_0$ is a dianalytic invariant.

For the study of the local trajectory structure, we will introduce special conformal parameters in terms of which the representation of an N-quadratic differential $\widetilde{\Phi}$ becomes simple.

In a sufficiently small neighborhood $\widetilde{U}$ of a point $\widetilde{z}_0 \in X$, with the local parameter $z = h(\widetilde{z})$, one can select a single valued branch $\Psi(z)$ of the function $\int \sqrt{\varphi(z)}\mathrm{d}z$ in the neighborhood U of z_0 and a single valued branch $\widehat{\Psi}(w)$ of the function $\int \sqrt{\widehat{\varphi}(w)}\mathrm{d}w$ in the neighborhood $k(U)$ of $w_0 = k(z_0)$, where $q^{-1}(\widetilde{U}) = \{U, k(U)\}$; U and $k(U)$ are mapped homeomorphically by Ψ and $\widehat{\Psi}$ onto open sets V, respectively $\widehat{V}$. Then $\omega = \Psi(z)$ and $\widehat{\omega} = \widehat{\Psi}(k(z))$ are local parameters in U, respectively $k(U)$, in terms of which $\mathrm{d}\omega^2 = \varphi(z)\mathrm{d}z^2$, respectively $\mathrm{d}\widehat{\omega}^2 = \widehat{\varphi}(k(z))\mathrm{d}k(z)^2$. The parameters are uniquely determined up to a transformation $\omega \to \pm\,\omega+$ constant, respectively $\widehat{\omega} \to \pm\,\widehat{\omega}+$ constant (see [6]).

By definition, the natural parameter of φ near z_0 is ω and it will be denoted by $\omega = \mathrm{np}(\varphi, z_0; h)$ while the natural parameter of $\widehat{\varphi}$ near $k(z_0)$ is $\widehat{\omega}$ and it will be denoted by $\widehat{\omega} = \mathrm{np}(\widehat{\varphi}, k(z_0); h)$.

Theorem 4. *Under the same conditions as in Theorem* 3, *if* $\omega = \mathrm{np}(\varphi, z_0; h)$, $\widehat{\omega} = \mathrm{np}(\widehat{\varphi}, k(z_0); h)$, *respectively* $\omega' = \mathrm{np}(\varphi', w_0; h')$, $\widehat{\omega'} = \mathrm{np}(\widehat{\varphi'}, k(w_0); h')$ *are the corresponding natural parameters near* $\widetilde{z}_0 \in X$, *uniquely determined up to a transformation of type* $\omega \to \pm\,\omega+$ *constant, then*

$$\begin{cases} \mathrm{np}(\varphi, z_0; h) = \mathrm{np}(\varphi', w_0; h') \\ \mathrm{np}(\widehat{\varphi}, k(z_0); h) = \mathrm{np}(\widehat{\varphi'}, k(w_0); h') \end{cases}$$

if $h' \circ h^{-1}$ *is analytic,*

$$\begin{cases} \mathrm{np}(\varphi, z_0; h) = \mathrm{np}(\widehat{\varphi'}, k(w_0); h') \\ \mathrm{np}(\widehat{\varphi}, k(z_0); h) = \mathrm{np}(\varphi', w_0; h') \end{cases}$$

if $h' \circ h^{-1}$ *is antianalytic.*

Proof. The result is a direct consequence of Theorem 2 and the definition of the natural parameter of a quadratic differential on a Riemann surface. □

For every $\widetilde{z} \in X$, define the set $NP(\widetilde{z}) = \{(\omega, \widehat{\omega}; h) \mid (\widetilde{U}, h, V) \in A, \widetilde{z} \in \widetilde{U}\}$ and an equivalence relation "$\sim$" on $NP(\widetilde{z})$ by $(\omega, \widehat{\omega}; h) \sim (\omega', \widehat{\omega'}; h') \Leftrightarrow$ a) $\omega = \omega', \widehat{\omega} = \widehat{\omega'}$, if $h' \circ h^{-1}$ is an analytic mapping, or b) $\omega = \widehat{\omega'}$, $\widehat{\omega} = \omega'$, if $h' \circ h^{-1}$ is an antianalytic mapping.

Let $NP(\widetilde{z})/\sim$ be the set of the corresponding equivalence classes and $[\omega, \widehat{\omega}; h] \in NP(\widetilde{z})/\sim$ the equivalence class of $(\omega, \widehat{\omega}; h) \in NP(\widetilde{z})$.

The natural parameter of an N-meromorphic quadratic differential $\widetilde{\Phi}$ at $\widetilde{z}_0$ will be denoted by $\mathrm{np}(\widetilde{\Phi}, \widetilde{z}_0)$ and, by definition,

$$\mathrm{np}(\widetilde{\Phi}, \widetilde{z}_0) = [\mathrm{np}(\varphi, z_0; h), \mathrm{np}(\widehat{\varphi}, k(z_0); h); h]\,.$$

Remark 2. It follows from Theorem 4, that the natural parameter of an N-meromorphic quadratic differential $\widetilde{\Phi}$ at $\widetilde{z}_0$ is well defined, that is, it does not depend on the choice of the local parameter. Thus, the natural parameter of an N-meromorphic quadratic differential $\widetilde{\Phi}$ at $\widetilde{z}_0$ is a dianalytic invariant.

An horizontal arc with respect to the quadratic differential φ near z_0 is a smooth curve γ along which $\arg \mathrm{d}\omega^2 = \arg \varphi(z)\mathrm{d}z^2 = 0$. By definition, γ is an horizontal trajectory or simply a trajectory of the quadratic differential φ near z_0 if it is a maximal horizontal arc and we write $\gamma = \mathrm{tr}(\varphi, z_0; h)$, [6]. An horizontal arc with respect to the quadratic differential $\widehat{\varphi}$ near $k(z_0)$ is a smooth curve $\widehat{\gamma}$ along which $\arg \mathrm{d}\widehat{\omega}^2 = \arg \widehat{\varphi}(z)\mathrm{d}z^2 = 0$. By definition, $\widehat{\gamma}$ is an horizontal trajectory or simply a trajectory of the quadratic differential $\widehat{\varphi}$ near $k(z_0)$ if it is a maximal horizontal arc and we write $\widehat{\gamma} = \mathrm{tr}(\widehat{\varphi}, k(z_0); h)$.

THEOREM 5. *Under the same conditions as in Theorems* 3 *and* 4*, if* $\gamma = \mathrm{tr}(\varphi, z_0; h)$*,* $\widehat{\gamma} = \mathrm{tr}(\widehat{\varphi}, k(z_0); h)$*, respectively* $\gamma' = \mathrm{tr}(\varphi', w_0; h')$*,* $\widehat{\gamma'} = \mathrm{tr}(\widehat{\varphi'}, k(w_0); h')$*, near* $\widetilde{z}_0$ *are the corresponding trajectories, then*

$$\begin{cases} \mathrm{tr}(\varphi, z_0; h) = \mathrm{tr}(\varphi', w_0; h') \\ \mathrm{tr}(\widehat{\varphi}, k(z_0); h) = \mathrm{tr}(\widehat{\varphi'}, k(w_0); h') \end{cases}$$

if $h' \circ h^{-1}$ *is analytic, or*

$$\begin{cases} \mathrm{tr}(\varphi, z_0; h) = \mathrm{tr}(\widehat{\varphi'}, k(w_0); h') \\ \mathrm{tr}(\widehat{\varphi}, k(z_0); h) = \mathrm{tr}(\varphi', w_0; h') \end{cases}$$

if $h' \circ h^{-1}$ *is antianalytic.*

Proof. The result is a direct consequence of Theorem 2 and the definition of the trajectory of a quadratic differential on a Riemann surface. □

For every $\widetilde{z} \in X$ the set

$$\{(\gamma, \widehat{\gamma}; h) \mid \gamma, \widehat{\gamma} : [0,1] \to \mathbf{C} \text{ locally rectifiable curves}, (\widetilde{U}, h, V) \in A,\ \widetilde{z} \in \widetilde{U}\}$$

is denoted by $TR(\widetilde{z})$. Define an equivalence relation "$\sim$" on $TR(\widetilde{z})$ by $(\gamma, \widehat{\gamma}; h) \sim (\gamma', \widehat{\gamma'}; h') \Leftrightarrow$ a) $\gamma = \gamma'$, $\widehat{\gamma} = \widehat{\gamma'}$, if $h' \circ h^{-1}$ is an analytic mapping, or b) $\gamma = \widehat{\gamma'}$, $\widehat{\gamma} = \gamma'$, if $h' \circ h^{-1}$ is an antianalytic mapping.

Let $TR(\widetilde{z})/\sim$ be the set of the corresponding equivalence classes and $[\gamma, \widehat{\gamma}; h] \in TR(\widetilde{z})/\sim$ the equivalence class of $(\gamma, \widehat{\gamma}; h) \in TR(\widetilde{z})$.

The trajectory of a quadratic differential $\widetilde{\Phi}$ near $\widetilde{z}_0$ will be denoted by $\mathrm{tr}(\widetilde{\Phi}, \widetilde{z}_0)$ and, by definition,

$$\mathrm{tr}(\widetilde{\Phi}, \widetilde{z}_0) = [\mathrm{tr}(\varphi, z_0; h), \mathrm{tr}(\widehat{\varphi}, k(z_0); h); h]\,.$$

Remark 3. It follows from Theorem 5 that the trajectory of an N-meromorphic quadratic differential $\widetilde{\Phi}$ at $\widetilde{z}_0$ is well defined, that is, it does not depend on the choice of the local parameter. Thus, the trajectory of an N-meromorphic quadratic differential $\widetilde{\Phi}$ at $\widetilde{z}_0$ is a dianalytic invariant.

COROLLARY 1. *In the* (z)*-plane, the trajectory arcs are the lines which are mapped into horizontals in the* (ω)*-plane. Similarly, the trajectory arcs in the* (w)*-plane are the lines which are mapped into horizontals in* $(\widehat{\omega})$*-plane. The trajectory structure in the plane of the parameter* $\mathrm{np}(\widetilde{\Phi}, \widetilde{z}_0)$ *near* $\widetilde{z}_0$ *results from the* (z)*-plane, respectively* (w)*-plane, trajectory structure, by conformal mappings, coresponding to the nature of the points* z_0*, respectively* $k(z_0)$ (*see* [6]).

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